



Syntax & Semantics WiSe 2020/2021

Lecture 23: Predicate Logic

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Semantics Lectures

- ▶ **Lecture 21: Semantics Introduction**
Kroeger (2019), Chapters 1-2 and Chapters 5-6.
- ▶ **Lecture 22: Propositional Logic**
Kroeger (2019), Chapter 3-4.
Zimmermann & Sternefeld (2013), Chapter 7.
- ▶ **Lecture 23: Predicate Logic**
Kroeger (2019), Chapter 4.
Zimmermann & Sternefeld (2013), Chapter 10 (p. 244-258).
- ▶ **Lecture 24: Syntax & Semantics Interface**
Kearns (2011), Chapter 4.
Zimmermann & Sternefeld (2013), Chapter 4.

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Section 1: Recap of Lecture 22



Formal Definition: Extensions

“Let us denote the **extension** of an expression A by putting double brackets ‘ $\llbracket \ \rrbracket$ ’ around A , as is standard in semantics. The extension of an expression depends on the **situation s** talked about when uttering A ; so we add the index s to the closing bracket.”

Zimmermann & Sternefeld (2013), p. 85.

$\llbracket \text{Paul} \rrbracket_s = \text{Paul McCartney}^1$

$\llbracket \text{the biggest German city} \rrbracket_s = \text{Berlin}$

$\llbracket \text{table} \rrbracket_s = \{ \text{table}_1, \text{table}_2, \text{table}_3, \dots, \text{table}_n \}^2$

$\llbracket \text{sleep} \rrbracket_s = \{ \text{sleeper}_1, \text{sleeper}_2, \text{sleeper}_3, \dots, \text{sleeper}_n \}$

$\llbracket \text{eat} \rrbracket_s = \{ \langle \text{eater}_1, \text{eaten}_1 \rangle, \langle \text{eater}_2, \text{eaten}_2 \rangle, \dots, \langle \text{eater}_n, \text{eaten}_n \rangle \}$

¹Zimmermann & Sternefeld just put the full proper name in brackets here, Kroeger follows another convention and just put the first letter in lower case, e.g. $\llbracket p \rrbracket_s$.

²Kroeger (2019) uses upper case notation for both nouns and predicates, e.g. TABLE and SLEEP respectively.

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Formal Definition: Frege's Generalization

“The **extension of a sentence S** is its **truth value**, i.e., 1 if S is true and 0 if S is false.”

Zimmermann & Sternefeld (2013), p. 74.

S_1 : The African elephant is the biggest land mammal.

$\llbracket S_1 \rrbracket_s = 1$, with s being 21st century earth.

S_2 : The coin flip landed heads up.

$\llbracket S_2 \rrbracket_s = 1$, with s being a particular coin flip.

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Formal Definition: Proposition

“The **proposition expressed by a sentence** is the **set of possible cases [situations]** of which that sentence is true.”

Zimmermann & Sternefeld (2013), p. 141.

Coin-flip example:

situation	flip1	flip2
1	heads	heads
2	tails	tails
3	heads	tails
4	tails	heads

Sentence

S_1 : only one flip landed heads up

S_2 : all flips landed heads up

S_3 : flips landed at least once tails up

etc.

Proposition

$\llbracket S_1 \rrbracket = \{3, 4\}$

$\llbracket S_2 \rrbracket = \{1\}$

$\llbracket S_3 \rrbracket = \{2, 3, 4\}$

etc.

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Types of Sentences and Propositions

- ▶ **Analytic sentence (Tautology):** A sentence which is true in every situation, i.e. the proposition is a set which includes all possible situations.
Example: *Today is the first day of the rest of your life.*
- ▶ **Contradiction:** A sentence which is false in every situation, i.e. the proposition is an empty set.
Example: *Your children are not your children.*³
- ▶ **Synthetic sentence:** A sentence which is either true or false depending on the situation, i.e. the proposition is a non-empty subset of all possible situations.
Example: *The African elephant is the biggest land mammal.*

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³There are potentially situations in which this sentence might be true, depending on the different senses *child* might have.



Types of Inferences

There are (at least) **three types of inferences** that are relevant for analyzing sentence meanings:

- ▶ Inferences based on **content words**
- ▶ Inferences based on **logical words** (rather than content words)
- ▶ Inferences based on **quantifiers** (and logical words)

Kroeger (2019). Analyzing meaning, p. 56.

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Logical Word Inference

If inferences are drawn based purely on the **meaning of logical words** (operators), then the inference is generalizable to a potentially infinite number of premisses and conclusions. Note that we can replace the propositions by placeholders. Here, we are in the domain of **propositional logic**.

- (1) Premise 1: ***Either Joe is crazy or he is lying.***
Premise 2: ***Joe is not crazy.***

Conclusion: ***Therefore, Joe is lying.***

- (2) Premise 1: ***Either x or y.***
Premise 2: ***not x.***

Conclusion: ***Therefore, y.***

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Propositional Operators

We will here use the following operators:

Operator	Alternative Symbols	Name	English Translation
\neg	$\sim, !$	negation	<i>not</i>
\wedge	$., \&$	conjunction	<i>and</i>
\vee	$+, $	disjunction (inclusive <i>or</i>)	<i>or</i>
XOR	EOR, EXOR, $\oplus, \underline{\vee}$	exclusive <i>or</i>	<i>either ... or</i>
\rightarrow	\Rightarrow, \supset	material implication ⁴	<i>if ..., then</i>
\leftrightarrow	\Leftrightarrow, \equiv	material equivalence ⁵	<i>if, and only if ..., then</i>

Note: We will here assume that the English translations and the operators themselves are indeed equivalent in their meanings. However, in language usage, this might not actually be the case.

⁴aka *conditional*

⁵aka *biconditional*

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Truth Tables

In a **truth table** we identify the extensions of (declarative) sentences as truth values. In the notation typically used, the variables p and q represent such **truth values of sentences**.⁶ The left table below gives the notation according to Zimmermann & Sternefeld, the right table according to Kroeger. We will use the latter for simplicity.

$[[S_1]]_s$	$[[S_2]]_s$	$[[S_1]]_s \wedge [[S_2]]_s$	p	q	$p \wedge q$
1	1	1	T	T	T
1	0	0	T	F	F
0	1	0	F	T	F
0	0	0	F	F	F

⁶Kroeger (2019), p. 58 writes that p and q are variables that represent propositions. However, according to the definitions we have given above this is strictly speaking not correct.

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Conjunction

“In the same way, the operator \wedge ‘and’ can be defined by the truth table [below]. This table says that $p \wedge q$ (which is also sometimes written $p \& q$) is true just in case both p and q are true, and false in all other situations.”

Kroeger (2019). *Analyzing meaning*, p. 59.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(3) S_1 : *Peter is your child.*

$p \equiv \llbracket S_1 \rrbracket_s \in \{T, F\}$

(4) S_2 : *The moon is blue.*

$q \equiv \llbracket S_2 \rrbracket_s \in \{T, F\}$

$p \wedge q \equiv \llbracket S_1 \rrbracket_s \wedge \llbracket S_2 \rrbracket_s \in \{T, F\}$

Example: if the situation s is such that Peter *is* the child of the person referred to as *you*, but the moon is *not* blue, then

$p \wedge q \equiv \llbracket S_1 \rrbracket_s \wedge \llbracket S_2 \rrbracket_s = F$.

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Building Truth Tables for Complex Sentences

We will follow the following four steps to analyze the sentence below:

1. Identify the **logical words** and translate them into **logical operators**
2. **Decompose the sentence** into its component declarative parts and assign **variables** to them (i.e. p and q).
3. Translate the whole sentence into **propositional logic notation**
4. Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

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Example Sentence: *If the president is either crazy or he is lying, and it turns out he is lying, then he is not crazy.*



Beyond Propositional Logic

“The propositional logic outlined in this section is an important part of the logical metalanguage for semantic analysis, but it is not sufficient on its own because it is concerned only with **truth values** [of whole sentences]. We need a way to go beyond p and q , to represent the actual meanings of **the basic propositions** we are dealing with.”

Kroeger (2019). *Analyzing meaning*, p. 66.

Example Sentences (Set 1):

p : John is hungry.

q : John is smart.

r : John is my brother.

Example Sentences (Set 2):

p : John snores.

q : Mary sees John.

r : Mary gives George a cake.

Note: Propositional logic assigns variables (p , q , r) to whole declarative sentences, and hence is “blind” to the fact that the first set of sentences shares both the same subject, and the copula construction, whereas the second set of sentences uses predicates of different valencies and different subjects and objects.

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Beyond Propositional Logic

A second major limitation of propositional logic is that it cannot take into account **quantifications**, and hence cannot decide on the truth values of the classical syllogisms below.

- (5) Premise 1: **All** men are mortal.
Premise 2: Socrates is a man.

Conclusion: Therefore, Socrates is mortal.

- (6) Premise 1: Arthur is a lawyer.
Premise 2: Arthur is honest.

Conclusion: Therefore, **some (= at least one)** lawyer is honest.

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Section 2: Historical Notes



Historical Perspective

“In the Hellenistic period, and apparently independent of Aristotle’s achievements, the logician Diodorus Cronus and his pupil Philo (see the entry Dialectical school) worked out the beginnings of a logic that took **propositions, rather than terms**,⁷ as its basic elements. They influenced the second major theorist of logic in antiquity, the **Stoic Chrysippus (mid-3rd c.)**, whose main achievement is the **development of a propositional logic [...]**”

<https://plato.stanford.edu/archives/spr2016/entries/logic-ancient/>
(accessed 10/02/2021)

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← 3rd Century **Propositional Logic**

1810 1820 1830 1840 1850 1860 1870 1880 1890 1900 1910 1920 1930 1940 1950

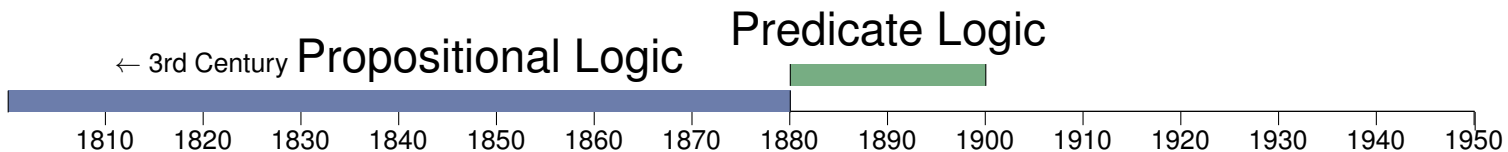
⁷A *term* here represents an object, a property, or an action like “Socrates” or “fall”, which cannot by itself be true or false. A *proposition* is then a combination of terms which can be assigned a truth value, e.g. “Socrates falls”.



Historical Perspective

“The first formulation of **predicate logic** can be found in Frege (1879); a similar system was developed independently by Peirce (1885). Modern versions radically differ from these ancestors in notation but not in their expressive means.”

Zimmermann & Sternefeld (2013), p. 244.



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“[...] fand ich ein Hindernis in der **Unzulänglichkeit der Sprache**, die bei aller entstehenden Schwerfälligkeit des Ausdruckes doch, je verwickelter die Beziehungen wurden, desto weniger die Genauigkeit erreichen liess, welche mein Zweck verlangte. Aus diesem Bedürfnisse ging der Gedanke der vorliegenden **Begriffsschrift** hervor.”

Frege (1879). Begriffsschrift: Eine der arithmetischen nachgebildete Formelsprache des reinen Denkens, p. X.

Translation: [...] I found the **inadequacy of language** to be an obstacle; no matter how unwieldy the expressions I was ready to accept, I was less and less able, as the relations became more and more complex, to attain the precision that my purpose required. This **deficiency** led me to the idea of the present **ideography**.



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Section 3: Basic Definitions



Logical Symbols

The following types of logical symbols are relevant for our analyses:

- ▶ **Logical operators (connectives)** equivalent to the ones defined in propositional logic: \neg , \wedge , \vee , \rightarrow , \leftrightarrow
- ▶ The **quantifier** symbols: \forall (universal quantifier), \exists (existential quantifier)
- ▶ An infinite set of variables: x , y , z , etc.⁸
- ▶ Parentheses ‘()’⁹

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⁸This set is called *Var* in Zimmermann & Sternefeld (2013), p. 244.

⁹Beware: In the propositional logic notation, we used parentheses ‘()’ for disambiguating the reading of a propositional logic expression as in $(p \rightarrow q) \wedge q$. However, in the predicate logic notation, parentheses can also have the function of denoting predicates (see below).



Logical Symbols: Quantifiers

“Standard predicate logic makes use of two quantifier symbols: the **Universal Quantifier** \forall , and the **Existential Quantifier** \exists . As the mathematical examples [below] illustrate, these quantifier symbols **must introduce a variable**, and this variable is said to be bound by the quantifier.”

Kroeger (2019) Analyzing meaning, p. 69.

Examples:

For all x it is the case that x plus x equals x times two.

There is some y for which y plus four equals y divided by three.

Quantifier notation:

$$\forall x(x+x = 2x)$$

$$\exists y(y+4 = y/3)$$

Note: Sometimes square brackets ‘[]’ are used here to illustrate the formulation that the quantifier scopes over. However, we use regular brackets.

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Non-Logical Symbols

The following types of **non-logical symbols** are relevant for our analyses:

- ▶ **Predicate symbols:** these are typically given as upper case letters, and reflect relations between n elements, where $n \geq 0$, and $n \in \mathbb{N}$ (i.e. natural numbers).¹⁰
- ▶ **Function symbols:** these are typically given with lower case letters (f , g , etc.), and take n variables as their arguments (similar to predicates), e.g. $f(x)$, $f(x, y)$, etc.

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¹⁰Zimmermann & Sternefeld (2013), p. 245 denote the set of all n -place predicates of a so-called *predicate logic lexicon* or *language* L as $PRED_{n,L}$.



Non-Logical Symbols: Predicates

Predicate symbols: these are typically given as upper case letters, and reflect relations between n elements, where $n \geq 0$, and $n \in \mathbb{N}$ (i.e. natural numbers). These are also called **n-ary** or **n-place predicate symbols**: $P(x)$, $P(x, y)$, $Q(x, y)$, etc.

Examples:

x snores

x is honest

x sees y

x gives y z

Predicate notation:

$P(x) \equiv \text{SNORE}(x)$

$Q(x) \equiv \text{HONEST}(x)$

$R(x, y) \equiv \text{SEE}(x, y)$

$S(x, y, z) \equiv \text{GIVE}(x, y, z)$

The single upper case letter notation is used by Zimmermann & Sternefeld (2013), the all capital notation is used by Kroeger (2019). Yet another notation involving primes (e.g. *snores'* was used earlier in the lecture following Müller (2019). In the following we will use the notation by Kroeger.

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Non-Logical Symbols: Functions

Function symbols are different from predicates since they do not denote a relation between the variables, but they **map the variables to unique values**. Importantly, a function with $n = 0$, i.e. zero valence, is called a **constant symbol** and denotes for example an individual or object.

Examples:

Socrates

Paris

the crocodile

father of x

Function notation:

s

p

c

$f(x)$

Note: s , j , p , and c are *constant symbols* here, i.e. strictly speaking zero valence functions, while $f(x)$ is a monovalent function. It is important to realize that while lower case letters are used for both *constant symbols* and *variables* (i.e. x), they represent different elements of predicate logic. The convention here is to use the *first letter of the respective name in lower case* as a constant symbol, while variables start at x .

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Section 4: Translating to Predicate Logic



N-place Predicates

Predicates with n arguments (*n-place, n-ary*) are simply dealt with by adding the *constant symbols* in parentheses of the predicate. The order of arguments is assumed to be the same as in the sentence.

- | | | |
|------|--|------------------------------|
| (7) | <i>Henry VIII snores.</i> | SNORE(h) |
| (8) | <i>Socrates is a man.</i> | MAN(s) ¹¹ |
| (9) | <i>Abraham admired Victoria.</i> | ADMIRE(a,v) ¹² |
| (10) | <i>Maria dio a Juan un libro.</i> | DAR(m,j,l) |
| (11) | <i>Jocasta is the mother of Oedipus.</i> | MOTHER_OF(j,o) ¹³ |

¹¹Remember that we said in the lectures on headedness, that sentences with a copula are a tricky case. Note that predicate logic treats the noun *man* here as the main predicate, rather than the copula.

¹²Inflections are not considered by predicate logic, hence, different persons, tenses, etc. are not reflected in the predicate logic formulation.

¹³While just the phrase *mother of oedipus* would have to be construed as a function, e.g. $f(o)$, the phrase *is mother of oedipus* is represented by a predicate.

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Predicates and Logical Operators

We can straightforwardly use **logical operators** in connection with **predicates**. For disambiguating the scope of logical operators (e.g. negation below) we will use normal parentheses.

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(12) *Abraham Lincoln was tall and homely.*

$TALL(a) \wedge HOMEPLY(a)$

(13) *Socrates was a bright man.*

$BRIGHT(s) \wedge MAN(s)$

(14) *Joe is neither honest nor competent.*

$\neg(HONEST(j) \vee COMPETENT(j))^{14}$
 \equiv
 $\neg HONEST(j) \wedge \neg COMPETENT(j)$

¹⁴Note that we here have to use the inclusive or \vee , rather than the exclusive XOR, since only the former allows for both predicates to be true, which is needed to negate them in a *neither...nor* statement.



Predicates of Predicates

Predicates can also be **embedded** into other predicates.

- (15) *Henry thinks that
Anne is beautiful.* THINK(h, BEAUTIFUL(a))
- (16) *Susan wants to marry Ringo.* WANT(s, MARRY(s,r))
- (17) *If you are honest, people will perceive you as competent.*
HONEST(y) → PERCEIVE(p, COMPETENT(y))

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Predicates and Quantifiers

Importantly, formulating predicates which involve **quantifications** requires the usage of particular logical operators, since quantifiers require variables, and the variables then need to be further linked to predicates via logical operators.

(18) *All students are weary.*

$\forall x(\text{STUDENT}(x) \rightarrow \text{WEARY}(x))$

lit. "For all x it is the case that if x is a student, then x is weary."

(19) *Some men snore.*

$\exists x(\text{MAN}(x) \wedge \text{SNORE}(x))$

lit. "There exists some x for which it is the case that x is a man and x snores."¹⁵

(20) *No crocodile is warm-blooded.*

$\neg \exists x(\text{CROCODILE}(x) \wedge \text{WARM-BLOODED}(x))$

lit. "It is not the case that there is some x for which x is a crocodile and x is warm-blooded."

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¹⁵Note that while the plural *men* suggests that we are talking about 2 or more individuals, the predicate logic formulation is valid for 1 or more individual(s).



Multi-Valent Predicates and Quantifiers

In the case of **multi-valent predicates** being combined with **quantifiers**, we typically have a combination of *variables* and *constant symbols* as arguments of the predicates. Indefinite noun phrases are typically translated using the existential quantifier.

(21) *Mary knows all the professors.*

$\forall x(\text{PROFESSOR}(x) \rightarrow \text{KNOW}(m,x))$

lit. “For all x it is the case that if x is a professor, then Mary knows x .”

(22) *Susan married a cowboy.*

$\exists x(\text{COWBOY}(x) \wedge \text{MARRY}(s,x))$

lit. “For some x it is the case that x is a cowboy and Susan married x .”¹⁶

(23) *Ringo lives in a yellow submarine.*

$\exists x(\text{YELLOW}(x) \wedge \text{SUBMARINE}(x) \wedge \text{LIVE_IN}(r,x))$

lit. “For some x it is the case that x is yellow, and x is a submarine, and that Ringo lives in x .”

¹⁶We might be tempted to drop the indefinite determiner and formulate just $\text{MARRY}(s,c)$. However, note that “a cowboy” and “the cowboy” are different, in the sense that the former is a not further defined individual in the set of cowboys, while the latter refers to a particular individual.

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Scope Ambiguities

“When a quantifier combines with another quantifier, with negation, or with various other elements [...], it can give rise to **ambiguities of scope**.”

Kroeger (2019). *Analyzing meaning*, p. 72.

(24) *Some man loves every woman.*

i. $\exists x(\text{MAN}(x) \wedge (\forall y(\text{WOMAN}(y) \rightarrow \text{LOVE}(x,y))))$

lit. “For some x it is the case that x is a man and (for all y it is the case that if y is a woman then x loves y).”

ii. $\forall y(\text{WOMAN}(y) \rightarrow (\exists x(\text{MAN}(x) \wedge \text{LOVE}(x,y))))$

lit. “For all y it is the case that if y is a woman then there is an x which is a man and loves y.”

(25) *All that glitters is not gold.*

i. $\forall x(\text{GLITTER}(x) \rightarrow \neg \text{GOLD}(x))$

lit. “For all x it is the case that if x glitters then x is not gold.”

ii. $\neg \forall x(\text{GLITTER}(x) \rightarrow \text{GOLD}(x))$

lit. “It is not the case for all x that if x glitters then x is gold.”

Note: In the first case the ambiguity is between whether the existential quantifier scopes over the universal quantifier, or the other way around. In the second example the ambiguity is whether the negation scopes over the universal quantifier or the other way around.

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Universal Instantiation

We can now translate the classical types of inferences (which are not covered by propositional logic) into predicate logic notation. Below is a classic inference called **universal instantiation**. By using a variable x bound by the universal quantifier (Premise 1), and then specifying this variable as a constant symbol (Premise 2), we adhere to a valid pattern of inference.

(26)	Premise 1: All men are mortal.	$\forall x(\text{MAN}(x) \rightarrow \text{MORTAL}(x))$
	Premise 2: <i>Socrates is a man.</i>	$\text{MAN}(s)$
<hr/>		
	Conclusion: <i>Therefore, Socrates is mortal.</i>	$\text{MORTAL}(s)$

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Existential Generalization

Another classic example is the so-called **existential generalization**. By asserting that two predicates are true for the same constant symbol (premise 1 and premise 2), we can generalize that there has to be a variable x for which both predicates hold.

(27) Premise 1: *Arthur is a lawyer.* $\text{LAWYER}(a)$
 Premise 2: *Arthur is honest.* $\text{HONEST}(a)$

Conclusion: *Therefore, **some (= at least one) lawyer is honest.***

$\exists x(\text{LAWYER}(x) \wedge \text{HONEST}(x))$

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Model Theory

“In order to develop and test a set of interpretive rules [...] it is important to provide very explicit descriptions for the test situations. As stated above, this kind of **description of a situation is called a model**, and must include two types of information: (i) **the domain**, i.e., the set of all individual entities in the situation; and (ii) the **denotation sets for the basic vocabulary items** [constant symbols, predicates] in the expressions being analyzed.”

Kroeger (2019). Analyzing meaning, p. 240.

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Example Domain

Let us assume our example **domain**, i.e. the so-called **universal set U** is a set of exactly three individuals that we can then use in further more complicated predicate logic expressions.

$U = \{\text{King Henry VIII, Anne Boleyn, Thomas Moore}\}$

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Example Denotations

Let us further assume the **denotation sets** of three predicates and three constant symbols. These denotation sets specify which individuals of U a particular expression can possibly denote.

$\llbracket \text{MAN} \rrbracket = \{\text{King Henry VIII, Thomas Moore}\}$

$\llbracket \text{WOMAN} \rrbracket = \{\text{Anne Boleyn}\}$

$\llbracket \text{SNORE} \rrbracket = \{\text{King Henry VIII}\}$

$\llbracket h \rrbracket = \{\text{King Henry VIII}\}$

$\llbracket a \rrbracket = \{\text{Anne Boleyn}\}$

$\llbracket t \rrbracket = \{\text{Thomas Moore}\}$

Note: We could further introduce *situations* s like in Zimmermann & Sternefeld's definitions and hence further divide the denotation sets according to situations.

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Example Model Evaluation

Based on our **example model**, consisting of the example domain and the example universal set, we can now evaluate the truth values of predicate logic expressions.

- ▶ **N-place predicates** are evaluated by whether the constant symbol, the pair of constant symbols, etc. is a member of the denotation set of the predicate.
- ▶ **Logical operators** are evaluated the same way as in propositional logic.
- ▶ **Quantifiers** are evaluated according to subset relations.

See Kroeger (2019). *Analyzing meaning*, p. 241.

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English sentence	logical form	interpretation	truth value
a. <i>Thomas More is a man.</i>	$MAN(t)$	$Thomas\ More \in \llbracket MAN \rrbracket$	T
b. <i>Anne Boleyn is a man or a woman.</i>	$MAN(a) \vee WOMAN(a)$	$Anne\ Boleyn \in (\llbracket MAN \rrbracket \cup \llbracket WOMAN \rrbracket)$	T
c. <i>Henry VIII is a man who snores.</i>	$MAN(h) \wedge SNORE(h)$	$Henry\ VIII \in (\llbracket MAN \rrbracket \cap \llbracket SNORE \rrbracket)$	T
d. <i>All men snore.</i>	$\forall x[MAN(x) \rightarrow SNORE(x)]$	$\llbracket MAN \rrbracket \subseteq \llbracket SNORE \rrbracket$	F
e. <i>No women snore.</i>	$\neg \exists x[WOMAN(x) \wedge SNORE(x)]$	$\llbracket WOMAN \rrbracket \cap \llbracket SNORE \rrbracket = \emptyset$	T



References



References

Kroeger, Paul R. (2019). *Analyzing meaning. An introduction to semantics and pragmatics*. Second corrected and slightly revised version. Berlin: Language Science Press.

Zimmermann, Thomas E. & Sternefeld, Wolfgang (2013). *Introduction to semantics. An essential guide to the composition of meaning*. Mouton de Gruyter.

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Thank You.

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