EBERHARD KARLS
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Faculty of Philosophy
General Linguistics

Syntax & Semantics WiSe 2020/2021

Lecture 22: Propositional Logic

08/02/2021, Christian Bentz

Overview

Semantics Lectures

- **Lecture 21**: Semantics Introduction
  Kroeger (2019), Chapters 1-2 and Chapters 5-6.
- **Lecture 22**: Propositional Logic
  Kroeger (2019), Chapter 3-4.
  Zimmermann & Sternefeld (2013), Chapter 7.
- **Lecture 23**: Predicate Logic
  Kroeger (2019), Chapter 4.
  Zimmermann & Sternefeld (2013), Chapter 10 (p. 244-258).
- **Lecture 24**: Syntax & Semantics Interface
  Kearns (2011), Chapter 4.
  Zimmermann & Sternefeld (2013), Chapter 4.

Q&As

Tutorial Week 8

In the solutions to Exercise 1 a) it says "Alternatively, the VP could also be split into $NP_{Subj/Obj}$ and V_{fin}." Which verb is this referring to?

Thanks for pointing this out. This is an error. It should be "VP could also be split into $NP_{Subj/Obj}$ and VP_{inf}". So this refers to VP which is headed by the non-finite verb.

Tutorial Week 8

> *In the solutions to Exercise 1b), do we not need to understand headedness in order to assign phrase symbols (e.g. "the neighbours" is an NP and not a DP)?*

This is an interesting point. Yes, in this particular case, arguably we need to understand headedness in order to assign phrase symbols. In fact, Goldberg (2006, p. 36) also makes reference to headedness when it comes to, for example, prepositional phrases. On the other hand, for constructions of sentences without verbs (Goldberg, 2006, p. 7) she proposes a "headless" analysis.

Tutorial Week 8

> *In the solution to Exercise 2 a) should we use* be_{fin} *or "is"?*

Strictly speaking, using "is" here wouldn't be wrong, since I asked you to come up with a construction for these three particular sentences. However, more generally the construction involves be_{fin}, since we can also use other finite forms of the copula to formulate grammatical sentences, e.g. *Mary was going crazy*.

Tutorial Week 8

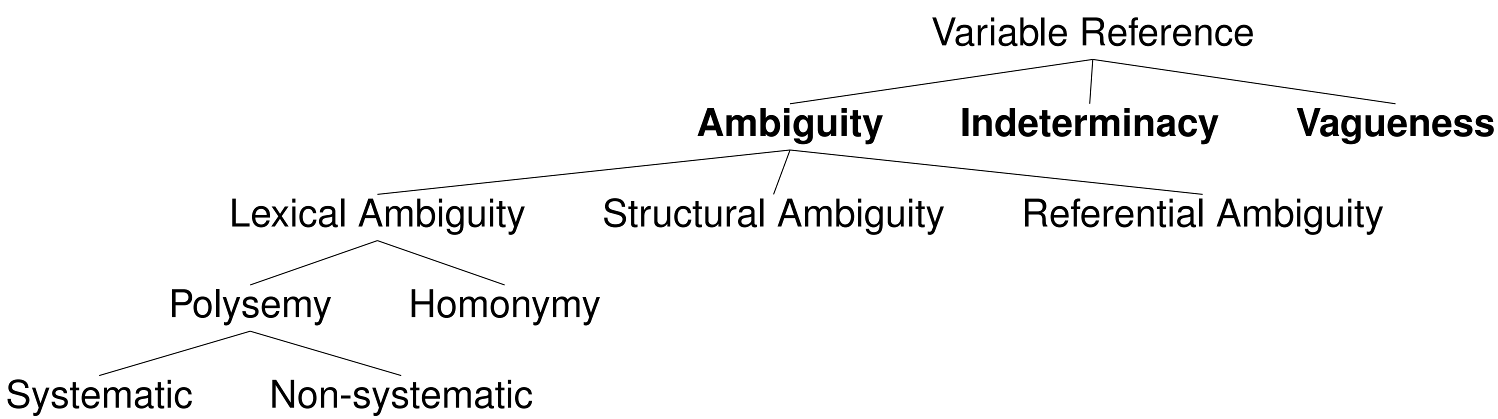
In the solution to Exercise 2 a) is the "to" of the to-infinitive included in the VP?

Yes, I think generally syntacticians would consider the "to" here to be part of the VP. I have clarified this in the answer: NP_{Subj} be_{fin} going $VP_{to-infinitive}$

Section 1: Recap of Lecture 21

Variable Reference

Even if we assume that reference between forms and meanings is generally possible (i.e. denotational semantics), then there is still the problem of **variable reference**, i.e. *ambiguity*, *indeterminacy* and *vagueness*.



Lexical Ambiguity

"It is possible for a single word to have more than one sense. [...] Words that have two or more senses are said to be **ambiguous** (more precisely, **polysemous** [...])."

Kroeger (2019). Analyzing meaning, p. 23

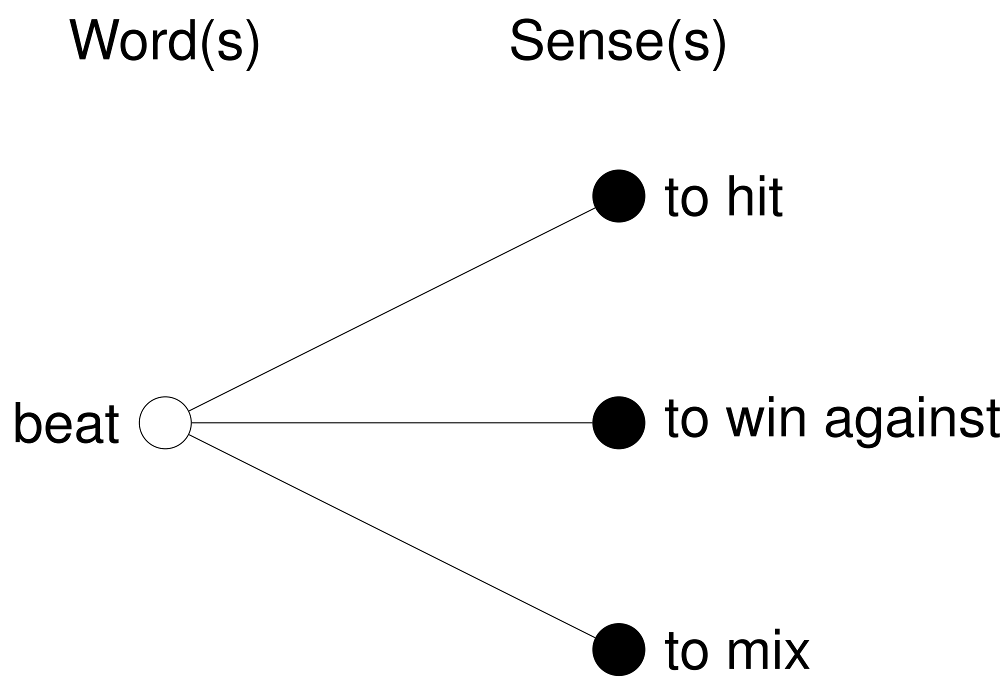
(1) A boiled egg is hard to *beat*.

(2) The farmer allows walkers to cross the field for free, but the bull *charges*.

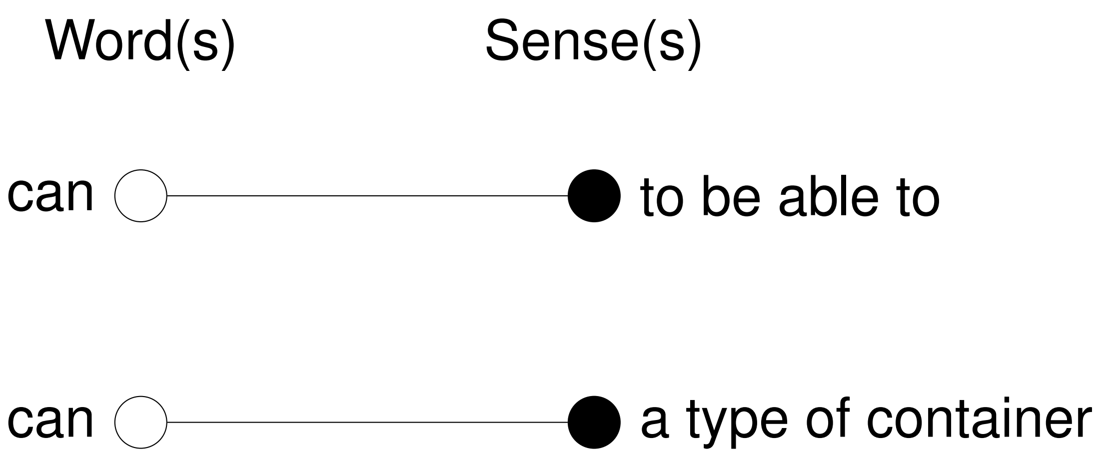
beat, verb
Sense 1: to strike or hit repeatedly
Sense 2: to win against
Sense 3: to mix thoroughly
etc.

https://dictionary.cambridge.org/dictionary/english-german/beat

Ambiguity (Polysemy)



Ambiguity (Homonymy)



Criteria for Polysemy

1. Semantic **feature/component sharing** (e.g. *foot* as bodypart and length measurement)
2. **Figurative extension** (e.g. *a road runs*)
3. Existence of a **primary sense** (e.g. the primary sense of *foot* is the body part)
4. **Etymology** (i.e. reconstructing the lexical sources, a method mostly used in dictionaries)

Kroeger (2019). Analyzing meaning, p. 90.

Indeterminacy

A type of variable reference, i.e. a word can have variability in its reference despite having a single defined sense. That is, the sense is **indeterminate** with regards to a particular dimension of meaning.

Kroeger (2019). Analyzing meaning, p. 81.

cousin, noun
Sense: a **son or daughter** of one's uncle or aunt.

https://dictionary.cambridge.org/dictionary/english-german/cousin

Note: The term *cousin* in English does not further specify the gender of the person referred to. Hence, it is indeterminate with regards to natural gender. In German, the natural gender is determined by the gender of the article and a suffix (*der Cousin*/ *die Cousin-e*).

Vagueness

A word is **vague** if the "limits of its possible denotations cannot be precisely defined."[1]

Kroeger (2019). Analyzing meaning, p. 81.

tall, adjective
Sense: (of people and thin or narrow objects such as buildings or trees) **higher than normal**

https://dictionary.cambridge.org/dictionary/english-german/tall

Note: The question here is "what is a *normal* height under which exact conditions?". In fact, this question can be answered precisely by statistics (e.g. more than two standard deviation above average), but humans do not necessarily use such words in a statistically precise way.

[1] Vagueness is sometimes also contrued as a cover term including indeterminacy as a sub-type. However, here the two are argued to be different concepts.

Indeterminacy versus Vagueness

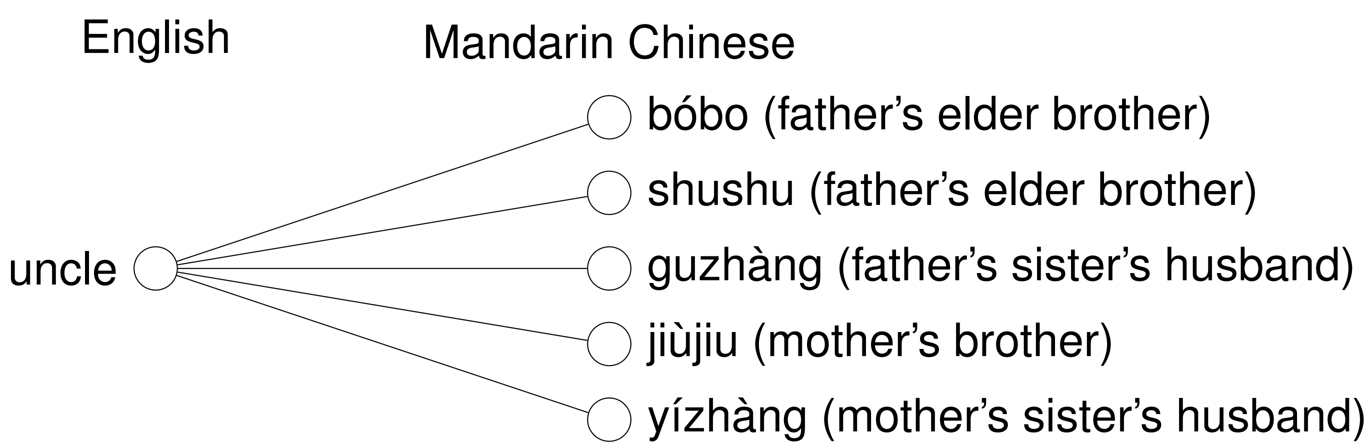
There are three charactersitics of vagueness which distinguish it from indeterminacy:

- **Context-dependence**: While the denotation of a vague word (e.g. *tall*) depends on the context (i.e. English Premier League Midfielder vs. Goalkeeper), the denotation of an indeterminate word does not depend on context (e.g. the family relationship indicated by *cousin* does not change according to context).
- **Borderline cases**: vague words display borderline cases due to their gradability (e.g. is 180cm tall for a EPL midfielder?), while for indeterminate words there is usually no disagreement (e.g. there is usually no disagreement about whether sb. is sb. else's cousin).
- **"Little-by-little" paradoxes**: due to the gradability of vague words, it is hard (impossible?) to determine when a certain denotation is justified (e.g. when exactly does a person with hair become a bald person?).

Indeterminacy versus Vagueness

"Another property which may distinguish vagueness from indeterminacy is the degree to which these properties are preserved in translation. Indeterminacy tends to be **language-specific**. There are many interesting and well-known cases where pairs of translation equivalents differ with respect to their degree of specificity."

Kroeger (2019). Analyzing meaning, p. 83.



Ambiguity vs. Vagueness/Indeterminacy

There are a range of tests proposed in the literature which are based on the fact that senses of ambiguous words are **antagonistic**, meaning that they cannot apply simultaneously:

- Zeugma Test
- Identity Test
- Sense Relations Test
- Contradiction Test

Kroeger (2019). Analyzing meaning, p. 84.

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Section 2: Basic Concepts

Three levels of meaning

1. **Word meaning**: Meaning assigned to individual words.
2. **Sentence meaning**: Meaning derived via combination of word meanings (compositional).
3. **Utterance meaning** ("speaker" meaning): "The term **utterance meaning** refers to the semantic content plus any pragmatic meaning created by the specific way in which the sentence gets used."

Kroeger (2019). Analyzing meaning, p.5.

Why use Formal Logic?

- We might (to some degree) **overcome** *ambiguity*, *vagueness*, *indeterminacy* inherent to language (if we want to).
- Logic provides precise rules and methods to determine the **relationships between meanings of sentences** (entailments, contradictions, paraphrase, etc.).
- Sytematically testing mismatches between logical inferences and speaker intuitions might help **determining the meanings of sentences**.
- Formal logic helps **modeling compositionality**.
- Formal logic is a **recursive system**, and might hence correctly model recursiveness in language.

Kroeger (2019). Analyzing meaning, p. 54.

Propositions

Proposition

"The meaning of a simple declarative sentence is called a **proposition**. A proposition is a claim about the world which may (in general) be true in some situations and false in others."

Kroeger (2019), p. 35.

"To know the meaning of a [declarative] sentence is to know what the world would have to be like for the sentence to be true."

Kroeger (2019), p. 35, citing Dowty et al. (1981: 4).

(3) *Mary snores.*

(4) *King Henry VIII snores.*

(5) *The unicorn in the garden snores.*

Formal Definition: Extension

Remember that in **denotational semantics** meaning is construed as the mapping between a given word and the real-world object it refers to (reference theory of meaning). More generally, words, phrases or sentences are said to have **extensions**, i.e. real-world situations they refer to.

Zimmermann & Sternefeld (2013), p. 71.

Type of expression	**Type of extension**	**Example**	**Extension of example**
proper name	individual	*Paul*	Paul McCartney
definite description	individual	*the biggest German city*	Berlin
noun	set of individuals	*table*	the set of tables
intransitive verb	set of individuals	*sleep*	the set of sleepers
transitive verb	set of pairs of individuals	*eat*	the set of pairs ⟨*eater*, *eaten*⟩
ditransitive verbs	set of triples of individuals	*give*	the set of triples ⟨*donator*, *recipient*, *donation*⟩

Formal Definition: Extensions

"Let us denote the **extension** of an expression *A* by putting double brackets '[[]]' around *A*, as is standard in semantics. The extension of an expression depends on the **situation *s*** talked about when uttering *A* ; so we add the index *s* to the closing bracket."

Zimmermann & Sternefeld (2013), p. 85.

$[\![\text{Paul}]\!]_s$ = Paul McCartney[2]
$[\![\text{the biggest German city}]\!]_s$ = Berlin
$[\![\text{table}]\!]_s = \{\text{table}_1, \text{table}_2, \text{table}_3, \ldots, \text{table}_n\}$[3]
$[\![\text{sleep}]\!]_s = \{\text{sleeper}_1, \text{sleeper}_2, \text{sleeper}_3, \ldots, \text{sleeper}_n\}$
$[\![\text{eat}]\!]_s = \{\langle\text{eater}_1, \text{eaten}_1\rangle, \langle\text{eater}_2, \text{eaten}_2\rangle, \ldots, \langle\text{eater}_n, \text{eaten}_n\rangle\}$

[2] Zimmermann & Sternefeld just put the full proper name in brackets here, Kroeger follows another convention and just puts the first letter in lower case, e.g. $[\![\text{p}]\!]_s$.

[3] Kroeger (2019) uses upper case notation for both nouns and predicates, e.g. TABLE and SLEEP respectively.

Formal Definition: Frege's Generalization

"The **extension of a sentence S** is its **truth value**, i.e., 1 if S is true and 0 if S is false."

Zimmermann & Sternefeld (2013), p. 74.

S_1: The African elephant is the biggest land mamal.
$[\![S_1]\!]_s = 1$, with s being 21st century earth.

S_2: The coin flip landed heads up.
$[\![S_2]\!]_s = 1$, with s being a particular coin flip.

Formal Definition: Proposition

"The **proposition expressed by a sentence** is the **set of possible cases [situations]** of which that sentence is true."
Zimmermann & Sternefeld (2013), p. 141.

Coin-flip example:

situation	flip1	flip2
1	heads	heads
2	tails	tails
3	heads	tails
4	tails	heads

Sentence	**Proposition**
S_1: only one flip landed heads up	$[\![S_1]\!] = \{3, 4\}$
S_2: all flips landed heads up	$[\![S_2]\!] = \{1\}$
S_3: flips landed at least once tails up	$[\![S_3]\!] = \{2, 3, 4\}$
etc.	etc.

Formal Definition: Proposition

We thus have the following definitions:

- The **proposition** expressed by a sentence is the set of possible situations of which that sentence is true.
- A sentence S is true of a possible situation s if and only if $[\![S]\!]_s = 1$.
- $[\![S]\!]$, in turn, is then the proposition expressed by S, such that: $[\![S]\!] \equiv \{s : [\![S]\!]_s = 1\}$
- A sentence S is true of a possible situation s if and only if $s \in [\![S]\!]$, formally: $[\![S]\!]_s = 1$ iff $s \in [\![S]\!]$.

Adopted from Zimmermann & Sternefeld (2013), p. 144.

Note: Zimmermann & Sternefeld extent the definition from situations s to **possible worlds** w in order to capture the totality of all possible cases rather than cases specific to a situation.

Types of Sentences and Propositions

- **Analytic sentence (Tautology)**: A sentence which is true in every situation, i.e. the proposition is a set which includes all possible situations.
  Example: *Today is the first day of the rest of your life*.
- **Contradiction**: A sentence which is false in every situation, i.e. the proposition is an empty set.
  Example: *Your children are not your children*.[4]
- **Synthetic sentence**: A sentence which is either true or false depending on the situation, i.e. the proposition is an non-empty subset of all possible situations.
  Example: *The African elephant is the biggest land mamal*.

[4]There are potentially situations in which this sentence might be true, depending on the different senses *child* might have.

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Inference

Inference

"[...] knowing that one fact or set of facts is true gives us an adequate basis for concluding that some other fact is also true. **Logic** is the **science of inference**."

Premisses: The facts which form the basis of the inference.
Conclusions: The fact which is inferred.

Kroeger (2019). Analyzing meaning, p. 55.

(6) Premise 1: *Either Joe is crazy or he is lying.*
Premise 2: *Joe is not crazy.*

Conclusion: *Therefore, Joe is lying*.

Syllogism

"An important variety of deductive argument in which a conclusion follows **from two or more premises**; especially the categorical syllogism."

http://www.philosophypages.com/dy/s9.htm#syl

Categorical Syllogism

"A logical argument consisting of **exactly three categorical propositions, two premises and the conclusion,** with a total of exactly three categorical terms, each used in only two of the propositions."

http://www.philosophypages.com/dy/c.htm#casyl

Note: The distinction between *syllogism* and *categorical syllogism* is typically dropped by logicians, and inferences drawn from premises are called syllogisms in general.

Types of Inference

There are (at least) **three types of inferences** that are relevant for analyzing sentence meanings:

- Inferences based on **content words**
- Inferences based on **logical words** (rather than content words)
- Inferences based on **quantifiers** (and logical words)

Kroeger (2019). Analyzing meaning, p. 56.

Content Word Inference

If inferences are drawn based purely on **content words**, then we are strictly speaking outside the domain of logic, since logic deals with generalizable patterns of inference, rather than ideosyncrasies of individual words and their meanings.

(7) Premise: *John **killed** the wasp.*

Conclusion: *Therefore, the wasp **died***.

Note: The validity of the inference here depends on our understanding and definition of the words *killed* and *died*. *Kill* is typically defined as "to cause sb. or sth. to die". Hence, the inference is valid.

Logical Word Inference

If inferences are drawn based purely on the **meaning of logical words** (operators), then the inference is generalizable to a potentially infinite number of premisses and conclusions. Note that we can replace the propositions by placeholders. Here, we are in the domain of **propositional logic**.

(8) Premise 1: ***Either*** *Joe is crazy* ***or*** *he is lying.*
Premise 2: *Joe is* ***not*** *crazy.*

Conclusion: ***Therefore***, *Joe is lying*.

(9) Premise 1: ***Either*** *x* ***or*** *y.*
Premise 2: ***not*** *x.*

Conclusion: ***Therefore***, *y*.

Quantifier Inference

If quantifiers are used (on top of other logical operators), pure propositional logic is not sufficient anymore. We are then in the domain of **predicate logic**.

(10) Premise 1: ***All*** *men are mortal.*
Premise 2: *Socrates is a man.*

Conclusion: *Therefore, Socrates is mortal.*

Section 3: Propositional Logic

Propositional Operators

We will here use the following operators:

Operator	Alternative Symbols	Name	English Translation
$\neg$	$\sim$, !	negation	*not*
$\wedge$	., &	conjunction	*and*
$\vee$	+, ||	disjunction (inclusive *or*)	*or*
XOR	EOR, EXOR, $\oplus$, $\veebar$	exclusive *or*	*either ... or*
$\rightarrow$	$\Rightarrow$, $\supset$	material implication[5]	*if ..., then*
$\leftrightarrow$	$\Leftrightarrow$, $\equiv$	material equivalence[6]	*if, and only if ..., then*

Note: We will here assume that the English translations and the operators themselves are indeed equivalent in their meanings. However, in language usage, this might not actually be the case.

[5] aka *conditional*
[6] aka *biconditional*

Truth Tables

In a **truth table** we identify the extensions of (declarative) sentences as truth values. In the notation typically used, the variables *p* and *q* represent such **truth values of sentences**.[7] The left table below gives the notation according to Zimmermann & Sternefeld, the right table according to Kroeger. We will use the latter for simplicity.

$[\![S_1]\!]_s$	$[\![S_2]\!]_s$	$[\![S_1]\!]_s \wedge [\![S_2]\!]_s$
1	1	1
1	0	0
0	1	0
0	0	0

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

[7] Kroeger (2019), p. 58 writes that *p* and *q* are variables that represent propositions. However, according to the definitions we have given above this is strictly speaking not correct.

Negation

"When we have said that p and ¬p must have opposite truth values in any possible situation, we have provided a definition of the negation operator; nothing needs to be known about the specific meaning of p."
Kroeger (2019). Analyzing meaning, p. 59.

p	¬p
T	F
F	T

(11) S_1: *Peter is your child.*
$p \equiv [\![S_1]\!]_s \in \{T, F\}$
$\neg p \equiv \neg[\![S_1]\!]_s \in \{T, F\}$
Example: if the situation *s* is such that Peter is *not* the child of the person referred to as *you*, then $p \equiv [\![S_1]\!]_s$ = F, and $\neg p \equiv \neg[\![S_1]\!]_s$ = T, otherwise the other way around.

Conjunction

"In the same way, the operator ∧ 'and' can be defined by the truth table [below]. This table says that p∧q (which is also sometimes written p&q) is true just in case both p and q are true, and false in all other situations."

Kroeger (2019). Analyzing meaning, p. 59.

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

(12) S_1: *Peter is your child.*
$p \equiv [\![S_1]\!]_s \in \{T, F\}$

(13) S_2: *The moon is blue.*
$q \equiv [\![S_2]\!]_s \in \{T, F\}$

$$p \wedge q \equiv [\![S_1]\!]_s \wedge [\![S_2]\!]_s \in \{T, F\}$$

Example: if the situation *s* is such that Peter *is* the child of the person referred to as *you*, but the moon is *not* blue, then $p \wedge q \equiv [\![S_1]\!]_s \wedge [\![S_2]\!]_s$ = F.

Disjunction (Inclusive *or*)

"The operator ∨ 'or' is defined by the truth table [below]. This table says that p∨q is true whenever either p is true or q is true; it is only false when both p and q are false. Notice that this *or* of standard logic is the *inclusive or*, corresponding to the English phrase *and/or*, because it includes the case where both p and q are true."

Kroeger (2019). Analyzing meaning, p. 60.

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

(14) S_1: *Peter is your child.*
$p \equiv [\![S_1]\!]_s \in \{T, F\}$

(15) S_2: *The moon is blue.*
$q \equiv [\![S_2]\!]_s \in \{T, F\}$

$$p \vee q \equiv [\![S_1]\!]_s \vee [\![S_2]\!]_s \in \{T, F\}$$

Example: if the situation *s* is such that Peter *is not* the child of the person referred to as *you*, but the moon *is* indeed blue, then $p \vee q \equiv [\![S_1]\!]_s \vee [\![S_2]\!]_s = T$.

Exclusive *or*

"[The table below] shows how we would define this exclusive "sense" of *or*, abbreviated here as XOR. The table says that p XOR q will be true whenever either p or q is true, but not both; it is false whenever p and q have the same truth value."

Kroeger (2019). Analyzing meaning, p. 60.

p	q	p XOR q
T	T	F
T	F	T
F	T	T
F	F	F

(16) S_1: *Peter is your child.*
$p \equiv [\![S_1]\!]_s \in \{T, F\}$

(17) S_2: *The moon is blue.*
$q \equiv [\![S_2]\!]_s \in \{T, F\}$

$$p \text{ XOR } q \equiv [\![S_1]\!]_s \text{ XOR } [\![S_2]\!]_s \in \{T, F\}$$

Example: if the situation *s* is such that Peter *is* the child of the person referred to as *you*, and the moon *is* indeed blue, then
$p \text{ XOR } q \equiv [\![S_1]\!]_s \text{ XOR } [\![S_2]\!]_s = F$.

Material Implication (Conditional)

"The material implication operator $\rightarrow$ is defined by the truth table [below]. (The formula p$\rightarrow$q can be read as *if p (then) q*, *p only if q*, or *q if p*.) The truth table says that p$\rightarrow$q is defined to be false just in case p is true but q is false; it is true in all other situations."
Note: p is called the *antecedent* here, and q the *consequent*.

Kroeger (2019). Analyzing meaning, p. 61.

p	q	p $\rightarrow$ q
T	T	T
T	F	F
F	T	T
F	F	T

(18) S_1: *Peter is your child.*
p $\equiv [\![S_1]\!]_s \in \{T, F\}$

(19) S_2: *The moon is blue.*
q $\equiv [\![S_2]\!]_s \in \{T, F\}$

$$p \rightarrow q \equiv [\![S_1]\!]_s \rightarrow [\![S_2]\!]_s \in \{T, F\}$$

Example: if the situation *s* is such that Peter *is* the child of the person referred to as *you*, but the moon *is not* blue, then $p \rightarrow q \equiv [\![S_1]\!]_s \rightarrow [\![S_2]\!]_s$ = F. In all other situations, it is T.

Material Equivalence (Biconditional)

"The formula p↔q (read as p *if and only if* q) is a short-hand or abbreviation for: (p→q) ∧ (q→p). The **biconditional** operator is defined by the truth table [below]."

Kroeger (2019). Analyzing meaning, p. 61.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(20) S_1: *Peter is your child.*
$p \equiv [\![S_1]\!]_s \in \{T, F\}$

(21) S_2: *The moon is blue.*
$p \equiv [\![S_2]\!]_s \in \{T, F\}$

$$p \leftrightarrow q \equiv ([\![S_1]\!]_s \leftrightarrow [\![S_2]\!]_s) \in \{T, F\}$$

Example: if the situation *s* is such that Peter *is* the child of the person referred to as *you*, and the moon *is* blue, or if both is *not* the case, then $p \leftrightarrow q \equiv [\![S_1]\!]_s \leftrightarrow [\![S_2]\!]_s$ = T. In all other situations, it is F.

Building Truth Tables for Complex Sentences

We will follow the following four steps to analyze the sentence below:

1. Identify the **logical words** and translate them into **logical operators**
2. **Decompose the sentence** into its component declarative parts and assign **variables** to them (i.e. p and q).
3. Translate the whole sentence into **propositional logic notation**
4. Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

Example Sentence: *If the president is either crazy or he is lying, and it turns out he is lying, then he is not crazy.*

First Step

Identify the **logical words** and translate them into **logical operators**.

If the president is **either** crazy **or** he is lying, **and** it turns out he is lying, **then** he is **not** crazy.

- if ... then: $\rightarrow$ (material implication)
- either ... or: XOR (exclusive *or*)
- and: $\wedge$ (conjunction)
- not: $\neg$ (negation)

Second Step

Decompose the sentence into its component declarative parts and assign **variables** to them (i.e. p and q).

If **the president is** either **crazy** or he **is lying**, and it turns out he is lying, then he is not crazy.

- p: the president is crazy
- q: the president is lying

Note: We make the assumption here that the pronoun *he* refers back to the NP introduced earlier in the discourse, i.e. *the president*.

Third Step

Translate the whole sentence into **propositional logic notation**.

If the president is either crazy or he is lying, and it turns out he is lying, then he is not crazy.

- p: the president is crazy
- $\neg$p: the president is not crazy
- q: the president is lying
- p XOR q: the president is either crazy or he is lying
- $\wedge$ q: and the president is lying
- $\rightarrow$: if the president ... then the president ...

Note: We have to break statements down to simple declarative sentences by ignoring such formulations as *it turns out*. We also have to understand that the XOR and $\wedge$ statements are "embedded" in the $\rightarrow$ statement.

Overall result: ((p XOR q) $\wedge$ q) $\rightarrow$ $\neg$p

Fourth Step

Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

$$((\mathbf{p} \text{ XOR } \mathbf{q}) \wedge q) \rightarrow \neg p$$

p	q
T	T
T	F
F	T
F	F

Fourth Step

Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

$$((\mathbf{p\ XOR\ q}) \wedge q) \rightarrow \neg p$$

p	q	p XOR q
T	T	F
T	F	T
F	T	T
F	F	F

Fourth Step

Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

$$((\mathbf{p\ XOR\ q}) \wedge \mathbf{q}) \rightarrow \neg p$$

p	q	p XOR q	(p XOR q) ∧ q
T	T	F	F
T	F	T	F
F	T	T	T
F	F	F	F

Fourth Step

Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

$$((p \text{ XOR } q) \wedge q) \rightarrow \neg \mathbf{p}$$

p	q	p XOR q	(p XOR q) ∧ q	¬ p
T	T	F	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	F	T

Fourth Step

Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

$$((\mathbf{p\ XOR\ q}) \wedge \mathbf{q}) \rightarrow \neg \mathbf{p}$$

p	q	p XOR q	(p XOR q) ∧ q	¬ p	((p XOR q) ∧ q) → ¬p
T	T	F	F	F	**T**
T	F	T	F	F	**T**
F	T	T	T	T	**T**
F	F	F	F	T	**T**

Section 4: Beyond Propositional Logic

Beyond Propositional Logic

"The propositional logic outlined in this section is an important part of the logical metalanguage for semantic analysis, but it is not sufficient on its own because it is concerned only with **truth values** [of whole sentences]. We need a way to go beyond p and q, to represent the actual meanings of **the basic propositions** we are dealing with."

Kroeger (2019). Analyzing meaning, p. 66.

Example Sentences (Set 1):

p: John is hungry.
q: John is smart.
r: John is my brother.

Example Sentences (Set 2):

p: John snores.
q: Mary sees John.
r: Mary gives George a cake.

Note: Propositional logic assigns variables (p, q, r) to whole declarative sentences, and hence is "blind" to the fact that the first set of sentences shares both the same subject, and the copula construction, whereas the second set of sentences uses predicates of different valencies and different subjects and objects.

Beyond Propositional Logic

A second major limitation of propositional logic is that it cannot take into account **quantifications**, and hence cannot decide on the truth values of the classical syllogisms below.

(22) Premise 1: ***All*** *men are mortal.*
Premise 2: *Socrates is a man.*

Conclusion: *Therefore, Socrates is mortal.*

(23) Premise 1: *Arthur is a lawyer.*
Premise 2: *Arthur is honest.*

Conclusion: *Therefore,* ***some (= at least one)*** *lawyer is honest.*

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Summary

Summary

- Formal logic is a tool to capture the **combinatoriality of meaning at the sentence level**.
- There are different **types of formal logical languages**, e.g. propositional logic and predicate logic.
- **Propositional logic** is able to deal with inferences based on **logical words** (and, or, not, etc.).
- It is limited by not including more fine-grained **predicate-level distinctions** and **quantifiers**.

References

References

Kroeger, Paul R. (2019). *Analyzing meaning. An introduction to semantics and pragmatics.* Second corrected and slightly revised version. Berlin: Language Science Press.

Zimmermann, Thomas E. & Sternefeld, Wolfgang (2013). *Introduction to semantics. An essential guide to the composition of meaning.* Mouton de Gruyter.

Thank You.

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