



# **Syntax & Semantics WS2019/2020**

## Lecture 23: Recapitulation of Semantics



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# Overview

Section 1: Semantics Introduction

Section 2: Word Meaning

Section 3: Propositional Logic

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# Semantics Lectures

- ▶ Lecture 18: Introduction to Semantics  
Kroeger (2019). Chapters 1-2.
- ▶ Lecture 19: Word Meaning  
Kroeger (2019). Chapter 5-6.
- ▶ Lecture 20: Propositional Logic  
Kroeger (2019). Chapter 3-4; and Zimmermann & Sternefeld Chapter 7.
- ▶ Lecture 21: Predicate Logic  
Kroeger (2019). Chapter 4; and Zimmermann & Sternefeld Chapter 10 (p. 244-258).
- ▶ Lecture 22: Syntax & Semantics Interface  
Kearns (2011). Semantics. Second Edition. Chapter 4.;  
Zimmermann & Sternefeld (2013), Chapter 4.

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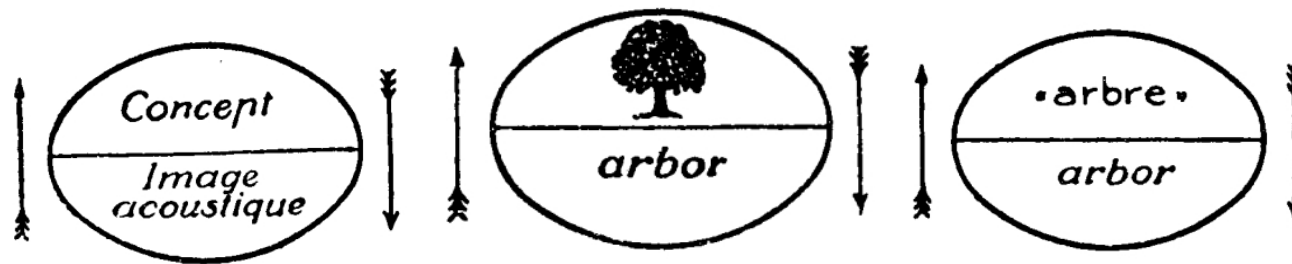
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# Section 1: Semantics Introduction



# The Roots

“Signifié et signifiant” at three levels:



Level 1: Abstract Relation

Level 2: Concrete Mapping (Denotation)

Level 3: Metalanguage (Translation)

Saussure (1995). *Cours de linguistique générale*, p. 99.

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# Arbitrariness

“For most words, the relation between the form (i.e. phonetic shape) of the word and its meaning is **arbitrary**. This is not always the case. Onomatopoeic words are words whose forms are intended to be imitations of the sounds which they refer to.”

Kroeger (2019). Analyzing meaning, p. 6.

## Arbitrary:

dog (English)  
shun (Armenian)  
cicing (Balinese)  
gae (Korean)  
aso (Tagalog)  
etc.

## Onomatopoeic:

bow-wow (English)  
haf-haf (Armenian)  
kong-kong (Balinese)  
mung-mung or  
wang-wang (Korean)  
etc.



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# However: Systematic Non-Arbitrariness

“[...] By analyzing word lists covering nearly **two-thirds of the world’s languages**, we demonstrate that a considerable proportion of **100 basic vocabulary items** carry strong associations with specific kinds of human speech sounds, occurring persistently across continents and linguistic lineages (linguistic families or isolates). Prominently among these relations, we find property words (“small” and i, “full” and p or b) and body part terms (“tongue” and l, “nose” and n).”

Blasi, Wichmann, Hammarström, Stadler, & Christiansen (2016). Sound-meaning association biases evidenced across thousands of languages.

Table 1. Summary of signals found in the ASJP database

Concept	Positive symbol	Negative symbol
Ash	u	—
Bite	k	—
Bone	k	y
Breasts	u m	a h r
Dog	s	t
Drink	—	a
Ear	k	—
Eye	—	a
Fish	a	—
Full	p b	—
Hear	N	—
Horn	k r	—
l	5	u p b t s r l
Knee	o u p k q	—
Leaf	b p l	—
Name	i	o p
Nose	u n	a
One	t n	—
Red	r	—
Round	r	—
Sand	s	—
Skin	—	m n
Small	i C	—
Star	z	—
Stone	t	—
Tongue	e E l	u k
Tooth	—	b m
Water	—	t
We	n	p l s
You	—	u o p t d q s r l

Positive and negative signals are those that have frequency significantly larger and smaller than expected.

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# Examples: Iconicity

Table 1. Some Iconic Associations Found in Ideophones across Languages [20,22]

Form	Meaning	Examples
Reduplication	Repetition, distribution	<i>goro</i> : <i>gorogoro</i> , 'one : multiple heavy objects rolling' (Japanese) <i>wùrùfù</i> : <i>wùrùfù-wùrùfù</i> , 'fluffy : fluffy here and there' (Siwu) <i>curuk-nu</i> : <i>curukcuruk-nu</i> , 'a sharp prick : many sharp pricks' (Tamil) <i>kpata</i> : <i>kpata kpata</i> , 'drop : scattered drops' (Ewe)
Vowel quality	Size, intensity	<i>katakata</i> : <i>kotokoto</i> , 'clattering : clattering (less noisy)' (Japanese) <i>pimbilii</i> : <i>pumbuluu</i> , 'small belly : enormous round belly' (Siwu) <i>ginigini</i> : <i>ginuginu</i> , 'tinkling : bell ringing' (Tamil) <i>legεε</i> : <i>logoo</i> , 'slim : fat' (Ewe)
Vowel lengthening	Length, duration	<i>haQ</i> : <i>haaQ</i> , 'short : long breath' (Japanese) <i>piQ</i> : <i>pīiQ</i> , 'tear short : long strip of cloth' (Japanese) <i>dzoro</i> : <i>dzoroo</i> 'long : very long' (Siwu)
Consonant voicing	Mass, weight	<i>koro</i> : <i>goro</i> , 'a light : heavy object rolling' (Japanese) <i>tsratsra</i> : <i>dzradzra</i> , 'a light : heavy person walking fast' (Siwu) <i>kputukpluu</i> : <i>gbudugbluu</i> , 'chunky : obese' (Ewe)

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Dingemanse, Blasi, Lupyan, Christiansen, & Monaghan (2015). Arbitrariness, iconicity, and systematicity in language.

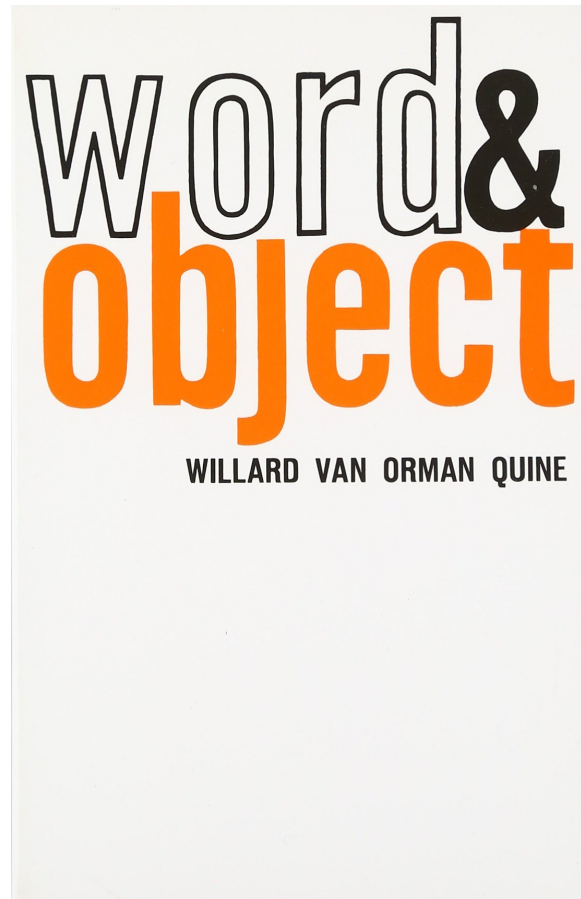




## Meaning as Reference

“What is relevant rather to our purposes is *radical translation*, i.e., translation of the language of a hitherto untouched people [...] The utterances first and most surely translated in such a case are ones keyed to present events that are conspicuous to the linguist and his informant. A rabbit scurries by, the native says ‘Gavagai’, and the linguist notes down the sentence ‘Rabbit’ or ‘Lo, a rabbit’) as tentative translation, subject to testing in further cases.”

Quine (1960). *Word and object*, p. 28.



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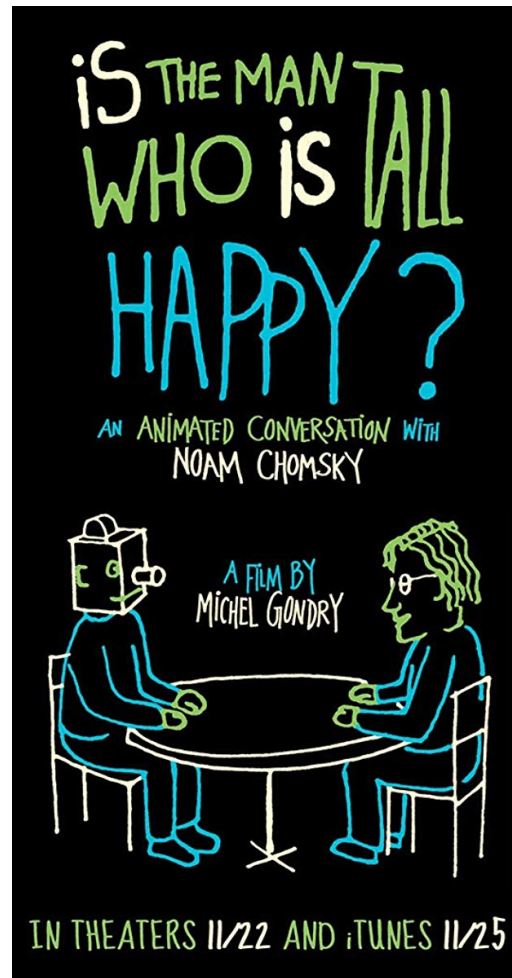
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## Against Reference: Words as Mental Representations

“It’s just a classic error that runs right through philosophy and psychology and linguistics right up to the moment. That’s the idea that words... say, meaning-bearing elements, like, say, “tree” or “person” or, you know, “John Smith” or anything... pick out something in the extramental world, something that a physicist could identify so that if I have a word... say, “cow”... it refers to something, and a, you know, scientist knowing nothing about my brain could figure out what counts as a cow. That’s just not true.”

Noam Chomsky



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# Semiotic Triangle (Triangle of Reference/Meaning)

“**Semiotics** is the study of the relationship between **signs and their meanings**. In this book we are interested in the relationship between forms and meanings in certain kinds of symbolic systems, namely human languages. The diagram is a way of illustrating how speakers use language to describe things, events, and situations in the world.”

Kroeger (2019). *Analyzing meaning*, p. 16.

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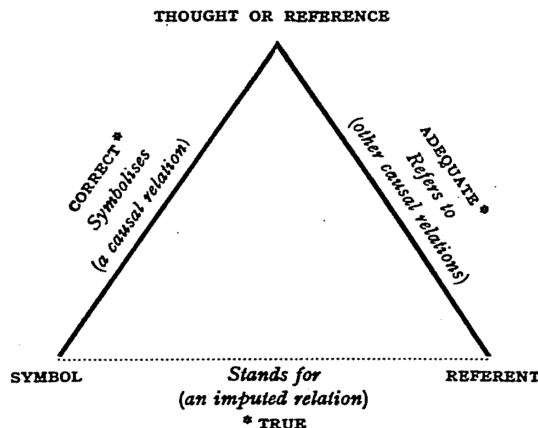
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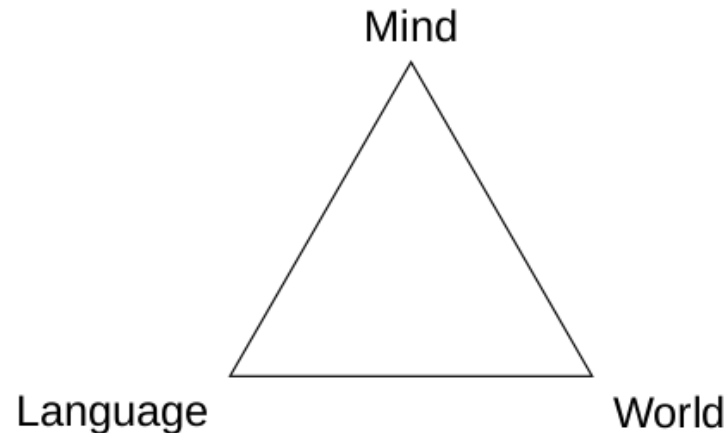
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Ogden & Richards (1923).  
*The meaning of meaning*, p. 11.



Kroeger (2019). *Analyzing meaning*, p. 16.



## Types of Referring Expressions

“A **referring expression** is an expression (normally some kind of noun phrase) which a speaker uses to refer to something. The identity of the referent is determined in different ways for different kinds of referring expressions.”

- ▶ Proper names
- ▶ “Natural kind” terms
- ▶ Deictic elements (indexicals)
- ▶ Anaphoric elements
- ▶ Definite descriptions
- ▶ Indefinite descriptions

Kroeger (2019). Analyzing meaning, p. 18.

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## Sense vs. Denotation

“The German logician Gottlob Frege (1848-1925) was one of the first people to demonstrate the importance of making this distinction. He used the German term *Sinn* (English **sense**) for those aspects of meaning which do **not depend on the context of use**, the kind of meaning we might look up in a dictionary.

Frege used the term *Bedeutung* (English **denotation**) for the other sort of meaning, which does **depend on the context**. The denotation of a referring expression, such as a proper name or definite NP, will normally be its referent.”

Kroeger (2019). Analyzing meaning, p. 21.

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## **Section 2: Word Meaning**



## Lexical Ambiguity

“It is possible for a single word to have more than one sense. [...] Words that have two or more senses are said to be **ambiguous** (more precisely, **polysemous** [...]).”

Kroeger (2019). Analyzing meaning, p. 23

- (1) A boiled egg is hard to *beat*.
- (2) The farmer allows walkers to cross the field for free, but the bull *charges*.

*beat*, verb

Sense 1: to strike or hit repeatedly

Sense 2: to win against

Sense 3: to mix thoroughly

etc.

<https://dictionary.cambridge.org/dictionary/english-german/beat>

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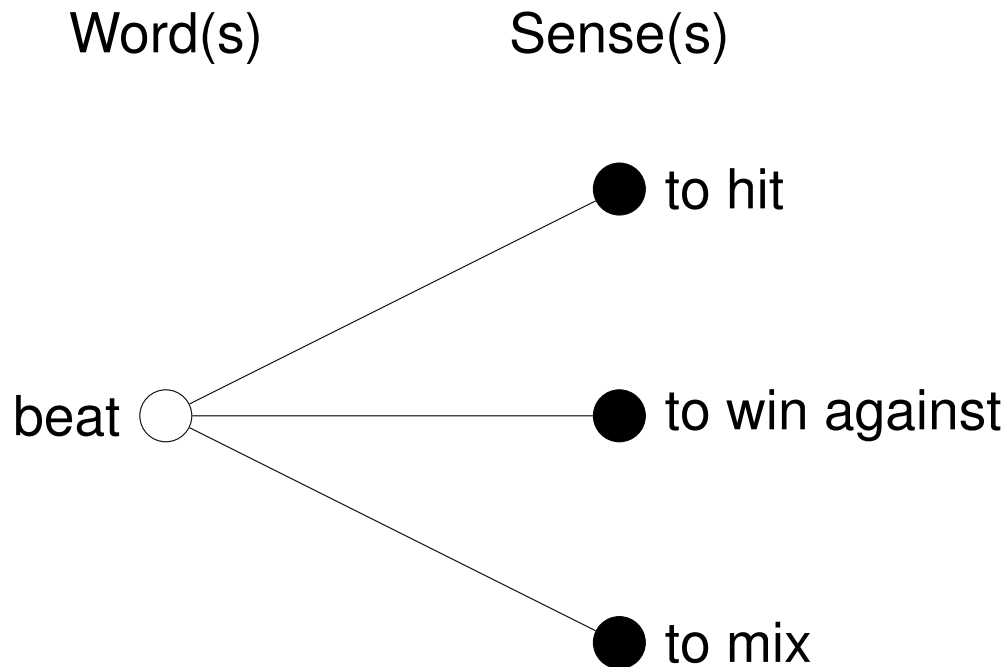
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# Ambiguity



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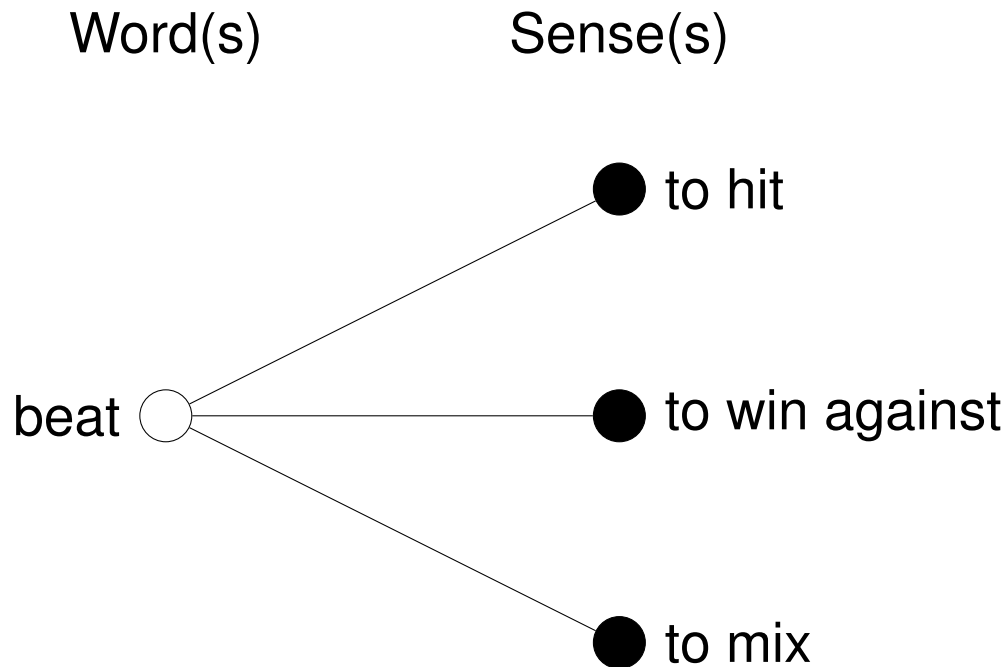
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# Ambiguity (Polysemy)



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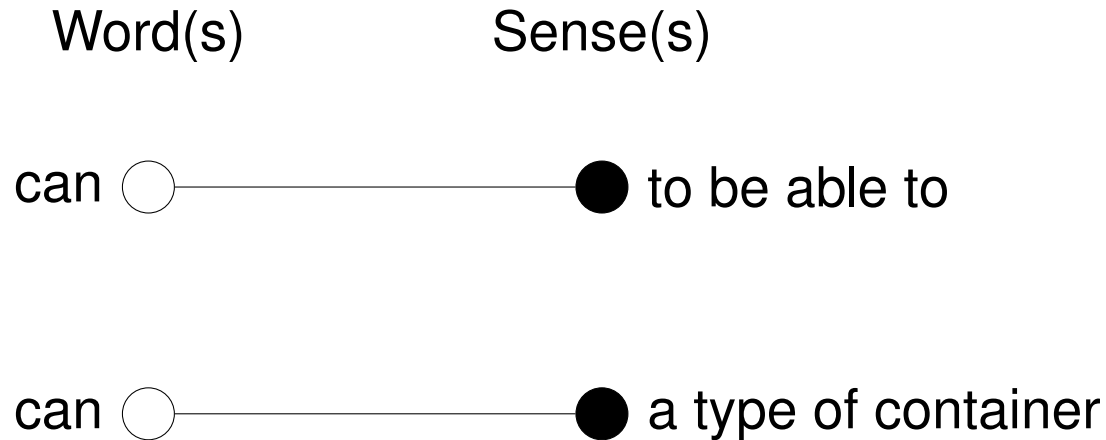
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# Ambiguity (Homonymy)



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## Criteria for Polysemy

1. Semantic **feature/component sharing** (e.g. *foot* as bodypart and length measurement)
2. **Figurative extension** (e.g. *a road runs*)
3. Existence of a **primary sense** (e.g. the primary sense of *foot* is the body part)
4. **Etymology** (i.e. reconstructing the lexical sources, a method mostly used in dictionaries)

Kroeger (2019). *Analyzing meaning*, p. 90.

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# Indeterminacy

A type of variable reference, i.e. a word can have variability in its reference despite having a single defined sense. That is, the sense is **indeterminate** with regards to a particular dimension of meaning.

Kroeger (2019). Analyzing meaning, p. 81.

*cousin*, noun

Sense: a **son or daughter** of one's uncle or aunt.

<https://dictionary.cambridge.org/dictionary/english-german/cousin>

Note: The term *cousin* in English does not further specify the gender of the person referred to. Hence, it is indeterminate with regards to natural gender. In German, the natural gender is determined by the gender of the article and a suffix (*der Cousin/ die Cousin-e*).

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# Vagueness

A word is **vague** if the “limits of its possible denotations cannot be precisely defined.”<sup>1</sup>

Kroeger (2019). Analyzing meaning, p. 81.

*tall*, adjective

Sense: (of people and thin or narrow objects such as buildings or trees)

**higher than normal**

<https://dictionary.cambridge.org/dictionary/english-german/tall>

Note: The question here is “what is a *normal* height under which exact conditions?”. In fact, this question can be answered precisely by statistics (e.g. more than two standard deviation above average), but humans do not necessarily use such words in a statistically precise way.

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<sup>1</sup>Vagueness is sometimes also construed as a cover term including indeterminacy as a sub-type. However, here the two are argued to be different concepts.

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## Indeterminacy versus Vagueness

“Another property which may distinguish vagueness from indeterminacy is the degree to which these properties are preserved in translation. Indeterminacy tends to be **language-specific**. There are many interesting and well-known cases where pairs of translation equivalents differ with respect to their degree of specificity.”

Kroeger (2019). *Analyzing meaning*, p. 83.

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## Ambiguity vs. Vagueness/Indeterminacy

There are a range of tests proposed in the literature which are based on the fact that senses of ambiguous words are **antagonistic**, meaning that they cannot apply simultaneously:

- ▶ Zeugma Test
- ▶ Identity Test
- ▶ Sense Relations Test
- ▶ Contradiction Test

Kroeger (2019). Analyzing meaning, p. 84.

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# The Advantages of Ambiguity

“We present a general information-theoretic argument that all efficient communication systems will be ambiguous, assuming that context is informative about meaning. We also argue that ambiguity allows for greater ease of processing by permitting efficient linguistic units to be re-used. Our results and theoretical analysis suggest that ambiguity is a functional property of language that allows for greater communicative efficiency.”

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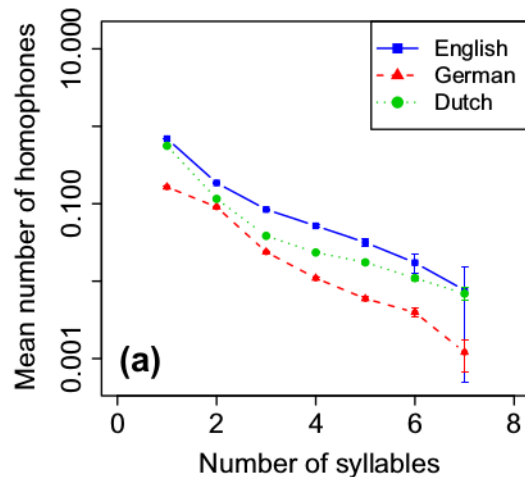
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Piantadosi et al. (2012). The communicative function of ambiguity in language.





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## Section 3: Propositional Logic



## Formal Definition: Extensions

“Let us denote the **extension** of an expression  $A$  by putting double brackets ‘ $\llbracket \ \rrbracket$ ’ around  $A$ , as is standard in semantics. The extension of an expression depends on the **situation  $s$**  talked about when uttering  $A$  ; so we add the index  $s$  to the closing bracket.”

Zimmermann & Sternefeld (2013), p. 85.

$\llbracket \text{Paul} \rrbracket_s = \text{Paul McCartney}^2$

$\llbracket \text{the biggest German city} \rrbracket_s = \text{Berlin}$

$\llbracket \text{table} \rrbracket_s = \{ \text{table}_1, \text{table}_2, \text{table}_3, \dots, \text{table}_n \}^3$

$\llbracket \text{sleep} \rrbracket_s = \{ \text{sleeper}_1, \text{sleeper}_2, \text{sleeper}_3, \dots, \text{sleeper}_n \}$

$\llbracket \text{eat} \rrbracket_s = \{ \langle \text{eater}_1, \text{eaten}_1 \rangle, \langle \text{eater}_2, \text{eaten}_2 \rangle, \dots, \langle \text{eater}_n, \text{eaten}_n \rangle \}$

<sup>2</sup>Zimmermann & Sternefeld just put the full proper name in brackets here, Kroeger follows another convention and just put the first letter in lower case, e.g.  $\llbracket p \rrbracket_s$ .

<sup>3</sup>Kroeger (2019) uses upper case notation for both nouns and predicates, e.g. TABLE and SLEEP respectively.

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## Formal Definition: Frege's Generalization

“The **extension of a sentence S** is its **truth value**, i.e., 1 if S is true and 0 if S is false.”

Zimmermann & Sternefeld (2013), p. 74.

S<sub>1</sub>: The African elephant is the biggest land mammal.

$\llbracket S_1 \rrbracket_s = 1$ , with  $s$  being 21st century earth.

S<sub>2</sub>: The coin flip landed heads up.

$\llbracket S_2 \rrbracket_s = 1$ , with  $s$  being a particular coin flip.

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## Formal Definition: Proposition

“The **proposition expressed by a sentence** is the **set of possible cases [situations]** of which that sentence is true.”

Zimmermann & Sternefeld (2013), p. 141.

Coin-flip example:

situation	flip1	flip2
1	heads	heads
2	tails	tails
3	heads	tails
4	tails	heads

### Sentence

$S_1$ : only one flip landed heads up

$S_2$ : all flips landed heads up

$S_3$ : flips landed at least once tails up

etc.

### Proposition

$\llbracket S_1 \rrbracket = \{3, 4\}$

$\llbracket S_2 \rrbracket = \{1\}$

$\llbracket S_3 \rrbracket = \{2, 3, 4\}$

etc.

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# Inference

“[...] knowing that one fact or set of facts is true gives us an adequate basis for concluding that some other fact is also true. **Logic** is the **science of inference**.”

**Premisses:** The facts which form the basis of the inference.

**Conclusions:** The fact which is inferred.

Kroeger (2019). Analyzing meaning, p. 55.

- (3) Premise 1: *Either Joe is crazy or he is lying.*  
Premise 2: *Joe is not crazy.*

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Conclusion: *Therefore, Joe is lying.*

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# Logical Word Inference

If inferences are drawn based purely on the **meaning of logical words** (operators), then the inference is generalizable to a potentially infinite number of premisses and conclusions. Note that we can replace the propositions by placeholders. Here, we are in the domain of **propositional logic**.

- (4) Premise 1: ***Either Joe is crazy or he is lying.***  
Premise 2: ***Joe is not crazy.***

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Conclusion: ***Therefore, Joe is lying.***

- (5) Premise 1: ***Either x or y.***  
Premise 2: ***not x.***

---

Conclusion: ***Therefore, y.***

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# Propositional Operators

We will here use the following operators:

Operator	Alternative Symbols	Name	English Translation
$\neg$	$\sim, !$	negation	<i>not</i>
$\wedge$	$., \&$	conjunction	<i>and</i>
$\vee$	$+,   $	disjunction (inclusive <i>or</i> )	<i>or</i>
XOR	EOR, EXOR, $\oplus, \underline{\vee}$	exclusive <i>or</i>	<i>either ... or</i>
$\rightarrow$	$\Rightarrow, \supset$	material implication <sup>4</sup>	<i>if ..., then</i>
$\leftrightarrow$	$\Leftrightarrow, \equiv$	material equivalence <sup>5</sup>	<i>if, and only if ..., then</i>

**Note:** We will here assume that the English translations and the operators themselves are indeed equivalent in their meanings. However, in language usage, this might not actually be the case.

<sup>4</sup>aka *conditional*

<sup>5</sup>aka *biconditional*

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# Truth Tables

In a **truth table** we identify the extensions of (declarative) sentences as truth values. In the notation typically used, the variables  $p$  and  $q$  represent such **truth values of sentences**.<sup>6</sup> The left table below gives the notation according to Zimmermann & Sternefeld, the right table according to Kroeger. We will use the latter for simplicity.

$[[S_1]]_s$	$[[S_2]]_s$	$[[S_1]]_s \wedge [[S_2]]_s$	$p$	$q$	$p \wedge q$
1	1	1	T	T	T
1	0	0	T	F	F
0	1	0	F	T	F
0	0	0	F	F	F

<sup>6</sup>Kroeger (2019), p. 58 writes that  $p$  and  $q$  are variables that represent propositions. However, according to the definitions we have given above this is strictly speaking not correct.

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# Building Truth Tables for Complex Sentences

We will follow the following four steps to analyze the sentence below:

1. Identify the **logical words** and translate them into **logical operators**
2. **Decompose the sentence** into its component declarative parts and assign **variables** to them (i.e. p and q).
3. Translate the whole sentence into **propositional logic notation**
4. Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

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Example Sentence: *If the president is either crazy or he is lying, and it turns out he is lying, then he is not crazy.*



## Beyond Propositional Logic

“The propositional logic outlined in this section is an important part of the logical metalanguage for semantic analysis, but it is not sufficient on its own because it is concerned only with **truth values** [of whole sentences]. We need a way to go beyond p and q, to represent the actual meanings of **the basic propositions** we are dealing with.”

Kroeger (2019). *Analyzing meaning*, p. 66.

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Example Sentences (Set 1):

p: John is hungry.

q: John is smart.

r: John is my brother.

Example Sentences (Set 2):

p: John snores.

q: Mary sees John.

r: Mary gives George a cake.

Note: Propositional logic assigns variables (p, q, r) to whole declarative sentences, and hence is “blind” to the fact that the first set of sentences shares both the same subject, and the copula construction, whereas the second set of sentences uses predicates of different valencies and different subjects and objects.



## Beyond Propositional Logic

A second major limitation of propositional logic is that it cannot take into account **quantifications**, and hence cannot decide on the truth values of the classical syllogisms below.

- (6) Premise 1: **All** men are mortal.  
Premise 2: Socrates is a man.

---

Conclusion: Therefore, Socrates is mortal.

- (7) Premise 1: Arthur is a lawyer.  
Premise 2: Arthur is honest.

---

Conclusion: Therefore, **some (= at least one)** lawyer is honest.

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## Section 4: Predicate Logic



## Historical Perspective

“The first formulation of predicate logic can be found in Frege (1879); a similar system was developed independently by Peirce (1885). Modern version radically differ from these ancestors in notation but not in their expressive means.”

Zimmermann & Sternefeld (2013), p. 244.

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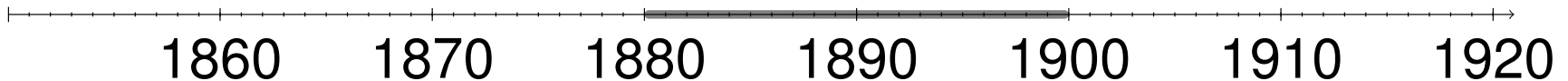
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## Logical Symbols

The following types of logical symbols are relevant for our analyses:

- ▶ **Logical operators (connectives)** equivalent to the ones defined in propositional logic:  $\neg$ ,  $\wedge$ ,  $\vee$ ,  $\rightarrow$ ,  $\leftrightarrow$
- ▶ The **quantifier** symbols:  $\forall$  (universal quantifier),  $\exists$  (existential quantifier)
- ▶ An infinite set of variables:  $x$ ,  $y$ ,  $z$ , etc.<sup>7</sup>
- ▶ Parentheses ‘()’ and brackets ‘[]’<sup>8</sup>

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<sup>7</sup>This set is called *Var* in Zimmermann & Sternefeld (2013), p. 244.

<sup>8</sup>Beware: In the propositional logic notation, we used parentheses ‘()’ for disambiguating the reading of a propositional logic expression as in  $(p \rightarrow q) \wedge q$ . However, in the predicate logic notation, parentheses can also have a different function (see below).



## Non-Logical Symbols

The following types of non-logical symbols are relevant for our analyses:

- ▶ **Predicate symbols:** these are typically given as upper case letters, and reflect relations between  $n$  elements, where  $n \geq 0$ , and  $n \in \mathbb{N}$  (i.e. natural numbers).<sup>9</sup>
- ▶ **Function symbols:** these are typically given with lower case letters ( $f, g$ , etc.), and take  $n$  variables as their arguments (similar to predicates), e.g.  $f(x)$ ,  $f(x, y)$ , etc.

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<sup>9</sup>Zimmermann & Sternefeld (2013), p. 245 denote the set of all  $n$ -place predicates of a so-called *predicate logic lexicon* or *language*  $L$  as  $PRED_{n,L}$ .



## Non-Logical Symbols: Predicates

**Predicate symbols:** these are typically given as upper case letters, and reflect relations between  $n$  elements, where  $n \geq 0$ , and  $n \in \mathbb{N}$  (i.e. natural numbers). These are also called **n-ary** or **n-place predicate symbols**:  $P(x)$ ,  $P(x, y)$ ,  $Q(x, y)$ , etc.

Examples:

$x$  snores

$x$  is honest

$x$  sees  $y$

$x$  gives  $y$   $z$

Predicate notation:

$P(x) \equiv \text{SNORE}(x)$

$Q(x) \equiv \text{HONEST}(x)$

$R(x,y) \equiv \text{SEE}(x,y)$

$S(x,y,z) \equiv \text{GIVE}(x,y,z)$

The single upper case letter notation is used by Zimmermann & Sternefeld (2013), the all capital notation is used by Kroeger (2019). Yet another notation involving primes (e.g. snore' was used earlier in the lecture following Müller (2019)). In the following we will use the notation by Kroeger.

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## Non-Logical Symbols: Functions

**Function symbols** are different from predicates since they do not denote a relation between the variables, but they **map the variables to unique values**. Importantly, a function with  $n = 0$ , i.e. zero valence, is called a **constant symbol** and denotes for example an individual or object.

Examples:

*Socrates*

*Paris*

*a crocodile*

*father of x*

Function notation:

*s*

*p*

*c*

*f(x)*

Note: *s*, *j*, *p*, and *c* are *constant symbols* here, i.e. strictly speaking zero valence functions, while *f(x)* is a monovalent function. It is important to realize that while lower case letters are used for both *constant symbols* and *variables* (i.e. *x*), they represent different elements of predicate logic. The convention here is to use the *first letter of the respective name in lower case* as a constant symbol, while variables start at *x*.

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## Multi-Valent Predicates and Quantifiers

In the case of **multi-valent predicates** being combined with **quantifiers**, we typically have a combination of *variables* and *constant symbols* as arguments of the predicates. Indefinite noun phrases are typically translated using the existential quantifier.

(8) *Mary knows all the professors.*

$\forall x[\text{PROFESSOR}(x) \rightarrow \text{KNOW}(m,x)]$

lit. "For all x it is the case that if x is a professor, then Mary knows x."

(9) *Susan married a cowboy.*

$\exists x[\text{COWBOY}(x) \wedge \text{MARRY}(s,x)]$

lit. "For some x it is the case that x is a cowboy and Susan married x."<sup>10</sup>

(10) *Ringo lives in a yellow submarine.*

$\exists x[\text{YELLOW}(x) \wedge \text{SUBMARINE}(x) \wedge \text{LIVE\_IN}(r,x)]$

lit. "For some x it is the case that x is yellow, and x is a submarine, and that Ringo lives in x."

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<sup>10</sup>Alternatively, we could drop the indefinite determiner and formulate just  $\text{MARRY}(s,c)$ . However, this is less precise and hence dispreferred.



## Scope Ambiguities

“When a quantifier combines with another quantifier, with negation, or with various other elements [...], it can give rise to ambiguities of scope.”

Kroeger (2019). *Analyzing meaning*, p. 72.

(11) *Some man loves every woman.*

i.  $\exists x[\text{MAN}(x) \wedge (\forall y[\text{WOMAN}(y) \rightarrow \text{LOVE}(x,y)])]$

lit. “For some x it is the case that x is a man and [for all y it is the case that y is a woman and x loves y].”

ii.  $\forall y[\text{WOMAN}(y) \rightarrow (\exists x[\text{MAN}(x) \wedge \text{LOVE}(x,y)])]$

lit. “For all y it is the case that if y is a woman then there is an x which is a man and x loves y.”

(12) *All that glitters is not gold.*

i.  $\forall x[\text{GLITTER}(x) \rightarrow \neg \text{GOLD}(x)]$

lit. “For all x it is the case that if x glitters then x is not gold.”

ii.  $\neg \forall x[\text{GLITTER}(x) \rightarrow \text{GOLD}(x)]$

lit. “It is not the case for all x that if x glitters then x is gold.”

Note: In the first case the ambiguity is between whether the existential quantifier scopes over the universal quantifier, or the other way around. In the second example the ambiguity is whether the negation scopes over the universal quantifier or the other way around.

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## Universal Instantiation

We can now translate the classical types of inferences (which are not covered by propositional logic) into predicate logic notation. Below is a classic inference called **universal instantiation**. By using a variable  $x$  bound by the universal quantifier (Premise 1), and then specifying this variable as a constant symbol (Premise 2), we adhere to a valid pattern of inference.

(13)	Premise 1: <b>All men are mortal.</b>	$\forall x[\text{MAN}(x) \rightarrow \text{MORTAL}(x)]$
	Premise 2: <i>Socrates is a man.</i>	$\text{MAN}(s)$
<hr/>		
	Conclusion: <i>Therefore, Socrates is mortal.</i>	$\text{MORTAL}(s)$

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## Existential Generalization

Another classic example is the so-called **existential generalization**. By asserting that two predicates are true for the same constant symbol (premise 1 and premise 2), we can generalize that there has to be a variable  $x$  for which both predicates hold.

- (14) Premise 1: *Arthur is a lawyer.*     $LAWYER(a)$   
 Premise 2: *Arthur is honest.*     $HONEST(a)$

---

Conclusion: *Therefore, **some (= at least one) lawyer is honest.***

$\exists x[LAWYER(x) \wedge HONEST(x)]$

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## Example Model Evaluation

Based on our **example model**, consisting of the example domain and the example universal set, we can now evaluate the truth values of predicate logic expressions. One-place predicates are evaluated by whether the constant symbol is a member of the denotation set of the predicate. Logical operators are evaluated the same way as in propositional logic. Quantifiers are evaluated according to subset relations.

See Kroeger (2019). *Analyzing meaning*, p. 241.

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English sentence	logical form	interpretation	truth value
a. <i>Thomas More is a man.</i>	$\text{MAN}(t)$	$\text{Thomas More} \in \llbracket \text{MAN} \rrbracket$	T
b. <i>Anne Boleyn is a man or a woman.</i>	$\text{MAN}(a) \vee \text{WOMAN}(a)$	$\text{Anne Boleyn} \in (\llbracket \text{MAN} \rrbracket \cup \llbracket \text{WOMAN} \rrbracket)$	T
c. <i>Henry VIII is a man who snores.</i>	$\text{MAN}(h) \wedge \text{SNORE}(h)$	$\text{Henry VIII} \in (\llbracket \text{MAN} \rrbracket \cap \llbracket \text{SNORE} \rrbracket)$	T
d. <i>All men snore.</i>	$\forall x[\text{MAN}(x) \rightarrow \text{SNORE}(x)]$	$\llbracket \text{MAN} \rrbracket \subseteq \llbracket \text{SNORE} \rrbracket$	F
e. <i>No women snore.</i>	$\neg \exists x[\text{WOMAN}(x) \wedge \text{SNORE}(x)]$	$\llbracket \text{WOMAN} \rrbracket \cap \llbracket \text{SNORE} \rrbracket = \emptyset$	T



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## Section 5: Syntax & Semantics Interface



## Valency in Semantics

“[...] one may detect an increasing complexity concerning the so-called **valency of verbs** [...] Corresponding to these types of predicates there are **three-place tuples (triples)**, **two-place tuples (pairs)** and **one-place tuples (individuals)**.”

*Parallelism between valency and type of extension:*

The extension of an  $n$ -place verb is always a set of  $n$ -tuples.

Zimmermann & Sternefeld (2013). Introduction to semantics, p. 72.

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Verb	Valency	Extension
<i>sleep</i>	monovalent	$[[\text{SLEEP}]]_s = \{\text{sleeper}_1, \text{sleeper}_2, \dots, \text{sleeper}_m\}$
<i>see</i>	bivalent	$[[\text{SEE}]]_s = \{\langle \text{seer}_1, \text{seen}_1 \rangle, \dots, \langle \text{seer}_m, \text{seen}_m \rangle\}$
<i>give</i>	trivalent	$[[\text{GIVE}]]_s = \{\langle \text{giver}_1, \text{receiver}_1, \text{given}_1 \rangle, \dots, \langle \text{giver}_m, \text{receiver}_m, \text{given}_m \rangle\}$

Note: We use  $m$  instead of  $n$  here as an index, in order to not confuse it with the  $n$  representing the valency.





## Filling of Arguments/Gaps

As the arguments of an  $n$ -place verb are “filled in”, the extensions change according to how many *components*<sup>11</sup> are in the tuples.<sup>12</sup>

Zimmermann & Sternefeld (2013). Introduction to semantics, p. 72.

Verb or VP	Valency	Extension
_ shows _ _	3	set of all triples $\langle a, b, c \rangle$ where a shows b c
_ shows the president _	2	set of all pairs $\langle a, c \rangle$ where a shows the president c
_ shows the president the Vatican Palace	1	set of all individuals (1-tuples) $\langle a \rangle$ where a shows the president the Vatican Palace
The Pope shows the president the Vatican Palace	0	set of all 0-tuples $\langle \rangle$ where the Pope shows the president the Vatican Palace

<sup>11</sup>Zimmermann & Sternefeld (2013), p. 67 point out that we speak of *components* of tuples (ordered lists), but *elements* of sets.

<sup>12</sup>Note: the individuals (constant symbols) are here given as  $a$ ,  $b$ , and  $c$ . In the Kroeger (2019) notation, we would use  $p_1$ ,  $p_2$ ,  $v$  (the first letter of the respective name).

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# Combinatoriality in Semantics

- (15) Kim sieh-t                      ein-en                      groß-en  
kim see-PRS.3SG DET.INDF-ACC.SG big-ACC.SG  
Baum  
tree.ACC.SG  
“Kim sees a big tree”  
 $\exists x[\text{TREE}(x) \wedge \text{SEE}(k,x)]$

In the example above, the meaning of the overall sentence arguably derives as a *combination* of the meanings of the individual parts.

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## Formal Composition

“**Compositional semantic theories** assume that the syntax and semantics work in parallel. For each *phrase structure rule* that combines two expressions into a larger phrase, there is a corresponding *semantic rule* which combines the meanings of the parts into the meaning of the newly formed expression.”

Kearns (2011). Semantics, p. 57.

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# Semantic Types

“Linguistic expressions are classified into their **semantic types** according to the kind of denotation they have. The two most basic denotation types are **type e**, the type of **entities**, and **type t**, the type of **truth values**.”

Kearns (2011). Semantics, p. 57.

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Type of expression	Type of extension	Semantic type	Example
proper name	individual (entity)	<b>e</b>	$[[\text{Paul}]]_s = \text{Paul McCartney}$
...	...	...	...
sentence	truth value	<b>t</b>	$[[\text{Paul is happy}]]_s \in \{0, 1\}$



## Functional Application

“[...] a function binds arguments together into a statement. From this insight, Frege proposed that all semantic composition is **functional application**. Functional application is just the combination of a function with an argument.”

Kearns (2011), p. 58.

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## Formal Definition

“We can define the following **combinatorial rule** for [...] typed expressions:

If  $\alpha$  is of type  $\langle b, a \rangle$  and  $\beta$  of type  $b$ , then  $\alpha(\beta)$  is of type  $a$ .

This type of combination is called **functional application**.”

Müller (2019), p. 188.



## Example: Recursive Application

$$\alpha(\beta) = a$$

$$\alpha = \langle b, a \rangle \quad \beta = b$$

**Note:** The **functional application** of the component  $b$  to the tuple  $\langle b, a \rangle$  is a mapping from  $b$  to  $a$  (this is how mathematical functions are defined, see also Kroeger (2019), p. 235 on relations and functions). For illustration, this might be thought of as an inference: the tuple expresses *if  $b$  then  $a$* .  $b$  expresses  *$b$  is the case*, hence we get  $a$ . Importantly, it is always the *left component* in a tuple that is the argument, and the *right component* is the outcome *value*.

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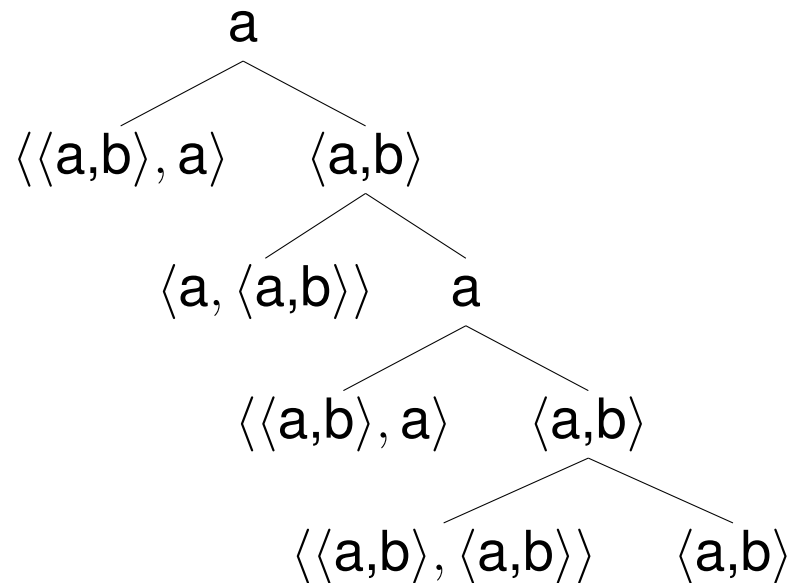
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## Example: Recursive Application



**Note:** Binarization does **not** mean that there are only a maximum of two components in each overall tuple. Instead there can be infinitely many 2-tuple embeddings. But each individual tuple can only have two components. Hence, we can build more complex semantic types out of the two basic types  $e$  and  $t$ .

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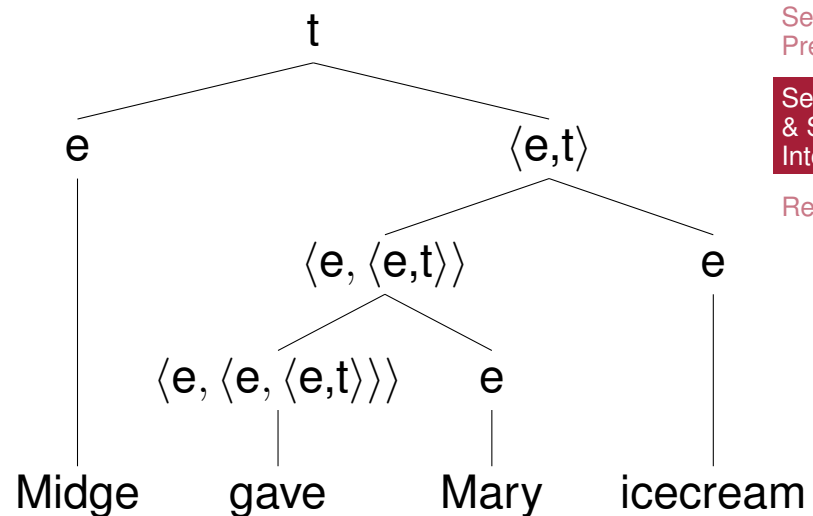
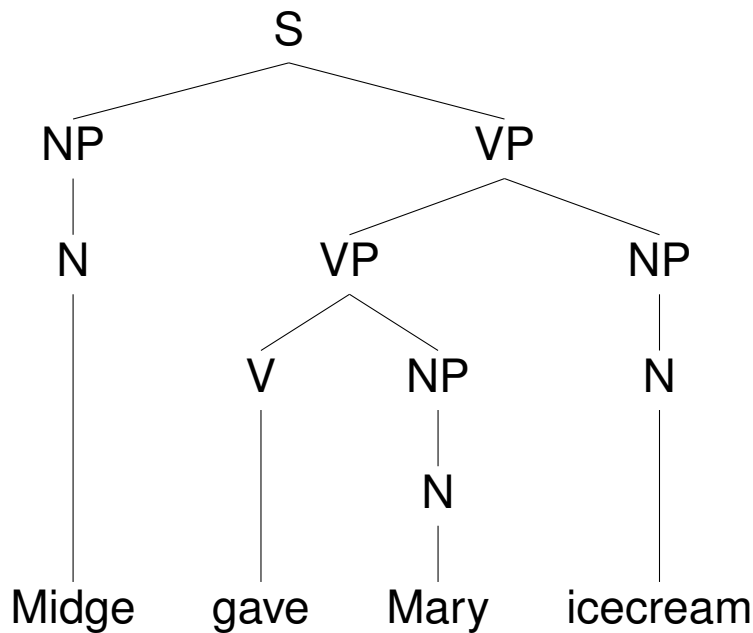
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## Semantic Types: Three-Place Predicates

A **ditransitive verb** requires **three arguments** to be filled in order to form a full sentence, hence it is of the **type**  $\langle e, \langle e, \langle e, t \rangle \rangle \rangle$ .



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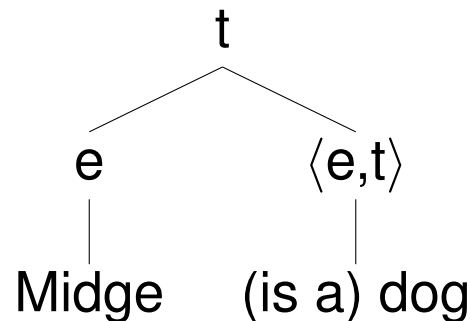
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## Semantic Types: Nouns

Common nouns are of type **type**  $\langle e, t \rangle$ . This might seem counterintuitive at first sight, but the idea here is that nouns are essentially like **one-place predicates**, in the sense that they require a concrete entity ( $e$ ) to form a basic existential statement (with a copular) which can be true or false.



Note: This corresponds to the predicate logic formulation  $DOG(m)$ , where the copular and the indefinite determiner are also dropped. As pointed out earlier in the lecture, the copular is a problematic and controversial element to analyze within syntactic theories, hence, the syntactic tree is not given here.

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# Summary: Semantic Types

## Type of Expression

## Semantic Type

Proper names

$e$

Sentences

$t$

Nouns

$\langle e, t \rangle$

Adjectives

$\langle e, t \rangle$

One-Place Predicates

$\langle e, t \rangle$

Two-Place Predicates

$\langle e, \langle e, t \rangle \rangle$

Three-Place Predicates

$\langle e, \langle e, \langle e, t \rangle \rangle \rangle$

Determiners

$\langle \langle e, t \rangle, e \rangle$

Adverbs

$\langle \langle e, t \rangle, \langle e, t \rangle \rangle$

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# Thank You.

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