



Faculty of Philosophy General Linguistics

Syntax & Semantics WiSe 2022/2023 Lecture 21: Predicate Logic

26/01/2023, Christian Bentz



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Semantics Lectures

- Lecture 18: Introduction to Semantics Kroeger (2019), Chapters 1-2.
 Lecture 19: Word Meaning Kroeger (2019), Chapter 5-6.
 Lecture 20: Propositional Logic Kroeger (2019), Chapter 3-4. Zimmermann & Sternefeld (2013), Chapter 7.
- Lecture 21: Predicate Logic Kroeger (2019), Chapter 4. Zimmermann & Sternefeld (2013), Chapter 10 (p. 244-258).

Lecture 22: Syntax & Semantics Interface Kearns (2011), Chapter 4. Zimmermann & Sternefeld (2013), Chapter 4. Section 1: Recap of Lecture 21

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Section 1: Recap of Lecture 21





Formal Definition: Extensions

"Let us denote the **extension** of an expression A by putting double brackets '[]]' around A, as is standard in semantics. The extension of an expression depends on the situation s talked about when uttering A; so we add the index s to the closing bracket."

Zimmermann & Sternefeld (2013), p. 85.

 $[Paul]_s = Paul McCartney^1$ [the biggest German city] s = Berlin $[table]_s = \{table_1, table_2, table_3, \dots, table_n\}^2$ $[sleep]_s = \{sleeper_1, sleeper_2, sleeper_3, \dots, sleeper_n\}$ $[eat]_s = \{ \langle eater_1, eaten_1 \rangle, \langle eater_2, eaten_2 \rangle, \dots, \langle eater_n, eaten_n \rangle \}$

¹Zimmermann & Sternefeld just put the full proper name in brackets here, Kroeger follows another convention and just put the first letter in lower case, e.g. $[p]_s$.

²Kroeger (2019) uses upper case notation for both nouns and predicates, e.g. TABLE and SLEEP respectively.

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Formal Definition: Frege's Generalization

"The **extension of a sentence S** is its **truth value**, i.e., 1 if S is true and 0 if S is false."

Zimmermann & Sternefeld (2013), p. 74.

S₁: The African elephant is the biggest land mamal. $[S_1]_s = 1$, with *s* being 21st century planet earth. $[S_1]_s = 0$, with *s* being planet earth.

S₂: The African elephant is the biggest mamal. $[S_2]_s = 0$, with *s* being 21st century planet earth. $[S_2]_s = 0$, with *s* being planet earth.

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Formal Definition: Proposition

"The proposition expressed by a sentence is the set of possible cases [situations] of which that sentence is true."

Zimmermann & Sternefeld (2013), p. 141.

Coin-flip example:

situation	flip1	flip2
1	heads	heads
2	tails	tails
3	heads	tails
4	tails	heads

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Sentence

- S₁: only one flip landed heads up
- S2: all flips landed heads up

S₃: flips landed at least once tails up etc.

Proposition

$$\begin{split} \llbracket S_1 \rrbracket &= \{3,4\} \\ \llbracket S_2 \rrbracket &= \{1\} \\ \llbracket S_3 \rrbracket &= \{2,3,4\} \\ etc. \end{split}$$



Types of Sentences and Propositions

Analytic sentence (Tautology): A sentence which is true in every situation, i.e. the proposition is a set which includes all possible situations.

Example: Today is the first day of the rest of your life.

Contradiction: A sentence which is false in every situation, i.e. the proposition is an empty set.

Example: Your children are not your children.³

Synthetic sentence: A sentence which is either true or false depending on the situation, i.e. the proposition is an non-empty subset of all possible situations.

Example: The African elephant is the biggest land mamal.

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³There are potentially situations in which this sentence might be true, depending on the different senses *child* might have.



Types of Inferences

There are (at least) **three types of inferences** that are relevant for analyzing sentence meanings:

- Inferences based on content words
- Inferences based on logical words (rather than content words)
- Inferences based on quantifiers (and logical words)

Kroeger (2019). Analyzing meaning, p. 56.

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Logical Word Inference

If inferences are drawn based purely on the **meaning of logical words** (operators), then the inference is generalizable to a potentially infinite number of premisses and conclusions. Note that we can replace the propositions by placeholders. Here, we are in the domain of **propositional logic**.

Premise 1: *Either* Joe is crazy or he is lying.
 Premise 2: Joe is not crazy.

Conclusion: *Therefore*, *Joe is lying*.

(2) Premise 1: *Either x or y.*Premise 2: *not x.*

Conclusion: Therefore, y.

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Propositional Operators

We will here use the following operators:

Operator	Alternative Symbols	Name	English Translation	Historical Notes
-	\sim , !	negation	not	Section 3: Basic
\wedge	., &	conjunction	and	Definitions
\vee	+,	disjunction (inclusive or)	or	Section 4:
XOR	EOR, EXOR, \oplus , \forall	exclusive <i>or</i>	either or	Predicate Logic
\rightarrow	\Rightarrow, \supset	material implication ⁴	if, then	Section 5:
\leftrightarrow	\Leftrightarrow,\equiv	material equivalence ⁵	if, and only if, then	Inference and Predicate Logic

Note: We will here assume that the English translations and the operators themselves are indeed equivalent in their meanings. However, in language usage, this might not actually be the case.

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⁴aka conditional ⁵aka *biconditional*



Truth Tables

In a **truth table** we identify the extensions of (declarative) sentences as truth values. In the notation typically used, the variables p and q represent such **truth values of sentences**.⁶ The left table below gives the notation according to Zimmermann & Sternefeld, the right table according to Kroeger. We will use the latter for simplicity.

$\llbracket S_1 \rrbracket_s$	$\llbracket S_2 \rrbracket_s$	$\llbracket S_1 rbracket_s \wedge \llbracket S_2 rbracket_s$	р	q	p∧q	
1	1	1	Т	Т	Т	
1	0	0	Т	F	F	
0	1	0	F	Т	F	
0	0	0	F	F	F	

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⁶Kroeger (2019), p. 58 writes that p and q are variables that represent propositions. However, according to the definitions we have given above this is strictly speaking not correct.



Conjunction

"In the same way, the operator \land 'and' can be defined by the truth table [below]. This table says that p \land q (which is also sometimes written p&q) is true just in case both p and q are true, and false in all other situations."

Kroeger (2019). Analyzing meaning, p. 59.

р	q	p∧q
Т	Т	Т
Т	F	F
F	Т	F
F	F	F

- (3) S₁: Peter is your child. $p \equiv [S_1]_s \in \{T, F\}$
- (4) S₂: The moon is blue. $q \equiv [S_2]_s \in \{T, F\}$

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 $p \land q \equiv [\![S_1]\!]_s \land [\![S_2]\!]_s \in \{T, F\}$ Example: if the situation *s* is such that

Peter *is* the child of the person referred to as *you*, but the moon is *not* blue, then $p \land q \equiv [S_1]_s \land [S_2]_s = F.$



Building Truth Tables for Complex Sentences

We will follow the following four steps to analyze the sentence below:

- 1. Identify the **logical words** and translate them into **logical operators**
- 2. **Decompose the sentence** into its component declarative parts and assign **variables** to them (i.e. p and q).
- 3. Translate the whole sentence into propositional logic notation
- 4. Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

Example Sentence: If the president is either crazy or he is lying, and it turns out he is lying, then he is not crazy.

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Beyond Propositional Logic

"The propositional logic outlined in this section is an important part of the logical metalanguage for semantic analysis, but it is not sufficient on its own because it is concerned only with truth values [of whole sentences]. We need a way to go beyond p and q, to represent the actual meanings of the basic propositions we are dealing with."

Kroeger (2019). Analyzing meaning, p. 66.

Example Sentences (Set 1):	Example Sentences (Set 2):
p: John is hungry.	p: John snores.
q: John is smart.	q: Mary sees John.

r: John is my brother.

- nn.
- r: Mary gives George a cake.

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Note: Propositional logic assigns variables (p, q, r) to whole declarative sentences, and hence is "blind" to the fact that the first set of sentences shares both the same subject, and the copula construction, whereas the second set of sentences uses predicates of different valencies and different subjects and objects.



Beyond Propositional Logic

A second major limitation of propositional logic is that it cannot take into account **quantifications**, and hence cannot decide on the truth values of the classical syllogisms below.

(5) Premise 1: *All men are mortal.* Premise 2: *Socrates is a man.*

Conclusion: Therefore, Socrates is mortal.

(6) Premise 1: *Arthur is a lawyer.* Premise 2: *Arthur is honest.*

Conclusion: Therefore, **some (= at least one)** lawyer is honest.

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Section 2: Historical Notes





Historical Perspective

"In the Hellenistic period, and apparently independent of Aristotle's achievements, the logician Diodorus Cronus and his pupil Philo (see the entry Dialectical school) worked out the beginnings of a logic that took propositions, rather than terms,⁷ as its basic elements. They influenced the second major theorist of logic in antiquity, the Stoic Chrysippus (mid-3rd c.), whose main achievement is the development of a propositional logic [...]"

https://plato.stanford.edu/archives/spr2016/entries/logic-ancient/



⁷A *term* here represents an object, a property, or an action like "Socrates" or "fall", which cannot by itself be true or false. A proposition is then a combination of terms which can be assigned a truth value, e.g. "Socrates falls".

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Historical Perspective

"The first formulation of **predicate logic** can be found in Frege (1879); a similar system was developed independently by Peirce (1885). Modern versions radically differ from these ancestors in notation but not in their expressive means."

Zimmermann & Sternefeld (2013), p. 244.



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"[...] fand ich ein Hindernis in der **Unzulänglichkeit der Sprache**, die bei aller entstehenden Schwerfälligkeit des Ausdruckes doch, je verwickelter die Beziehungen wurden, desto

Beziehungen wurden, desto weniger die Genauigkeit erreichen liess, welche mein Zweck verlangte. Aus diesem Bedürfnisse ging der Gedanke der vorliegenden **Begriffsschrift** hervor."

Frege (1879). Begriffsschrift: Eine der arithmetischen nachgebildete Formelsprache des reinen Denkens, p. X.

Translation: [...] I found the **inadequacy of language** to be an obstacle; no matter how unwieldy the expressions I was ready to accept, I was less and less able, as the relations became more and more complex, to attain the precision that my purpose required. This **deficiency** led me to the idea of the present **ideography**.

BEGRIFFSSCHRIFT,

EINE DER ARITHMETISCHEN NACHGEBILDETE

FORMELSPRACHE

11.i

MPRIW

DES REINEN DENKENS.

VON

D⁴⁻ GOTTLOB FREGE, privationenteen der mathematik an der universität jena. Section 1: Recap of Lecture 21

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HALLE ^A/S. VERLAG VON LOUIS NEBERT. 1879.





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Section 3: Basic Definitions



Logical Symbols

The following types of logical symbols are relevant for our analyses:

- ► Logical operators (connectives) equivalent to the ones defined in propositional logic: ¬, ∧, ∨, →, ↔
- ► The quantifier symbols: ∀ (universal quantifier), ∃ (existential quantifier)
- An infinite set of variables: x, y, z, etc.⁸
- Parentheses '()'⁹

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⁸This set is called *Var* in Zimmermann & Sternefeld (2013), p. 244. ⁹Beware: In the propositional logic notation, we used parentheses '()' for disambiguating the reading of a propositional logic expression as in $(p \rightarrow q) \land q$. However, in the predicate logic notation, parentheses can also have the function of denoting predicates (see below).



Logical Symbols: Quantifiers

"Standard predicate logic makes use of two quantifier symbols: the **Universal Quantifier** \forall , and the **Existential Quantifier** \exists . As the mathematical examples [below] illustrate, these quantifier symbols **must introduce a variable**, and this variable is said to be bound by the quantifier."

Kroeger (2019) Analyzing meaning, p. 69.

Examples:

For all x it is the case that x plus x equals x times two. There is some y for which y plus four equals y divided by three.

Note: Sometimes square brackets '[]' are used here to illustrate the formulation that the quantifier scopes over. However, we use regular brackets.

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Quantifier notation:

 $\forall x(x+x = 2x)$

 $\exists y(y+4 = y/3)$



Non-Logical Symbols

The following types of **non-logical symbols** are relevant for our analyses:

- ▶ **Predicate symbols**: these are typically given as upper case letters, and reflect relations between *n* elements, where n > 0, and $n \in \mathbb{N}$ (i.e. natural numbers).¹⁰
- ▶ **Function symbols**: these are typically given with lower case letters (*f*, *g*, etc.), and take *n* variables (with $n \ge 0$) as their arguments (similar to predicates), e.g. f(x), f(x, y), etc.

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Predicate Logic

¹⁰Zimmermann & Sternefeld (2013), p. 245 denote the set of all n-place predicates of a so-called *predicate logic lexicon* or *language L* as $PRED_{n,L}$.

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Non-Logical Symbols: Predicates

Predicate symbols: these are typically given as upper case letters, and reflect relations between *n* elements, where n > 0, and $n \in \mathbb{N}$ (i.e. natural numbers). These are also called **n-ary** or **n-place predicate symbols**: P(x), P(x, y), Q(x, y), etc.

Examples:

x snores

x is honest

x sees y

x gives y z

Predicate notation: $P(x) \equiv SNORE(x)$ $Q(x) \equiv HONEST(x)$ $R(x,y) \equiv SEE(x,y)$ $S(x,y,z) \equiv GIVE(x,y,z)$ Section 1: Recap of Lecture 21

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The single upper case letter notation is used by Zimmermann & Sternefeld (2013), the all capital notation is used by Kroeger (2019). Yet another notation involving primes (e.g. *snore*') was used earlier in the lecture following Müller (2019). In the following we will use the notation by Kroeger.



Non-Logical Symbols: Functions

Function symbols are different from predicates since they do not denote a relation between the variables, but they map the variables to **unique values**. Importantly, a function with n = 0, i.e. zero valence, is called a **constant symbol** and denotes for example an individual or object.

Examples:	Function notation:	Section 5: Inference and Predicate Logic
Socrates	S	Section 6:
Paris	р	Evaluating Predicate Logic
the crocodile	С	Expressions
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Note: s, j, p, and c are constant symbols here, i.e. strictly speaking zero valence functions, while f(x) is a monovalent function. It is important to realize that while lower case letters are used for both *constant symbols* and *variables* (i.e. x), they represent different elements of predicate logic. The convention here is to use the first letter of the respective name in lower case as a constant symbol, while variables start at x.

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Section 4: Translating to Predicate Logic



N-place Predicates

Predicates with *n* **arguments** (*n*-*place*, *n*-*ary*) are simply dealt with by adding the *constant symbols* in parentheses of the predicate. The order of arguments is assumed to be the same as in the sentence.

(7)	Henry VIII snores.	SNORE(h)
(8)	Socrates is a man.	MAN(s) ¹¹
(9)	Abraham admired Victoria.	ADMIRE(a,v) ¹²
(10)	Maria dio a Juan un libro.	DAR(m,j,l)
(11)	Jocasta is the mother of Oedipus.	MOTHER_OF(j,o) ¹

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¹¹Remember that we said in the lectures on headedness, that sentences with a copula are a tricky case. Note that predicate logic treats the noun *man* here as the main predicate, rather than the copula.

¹²Inflections are not considered by predicate logic, hence, different persons, tenses, etc. are not reflected in the predicate logic formulation.

¹³While just the phrase *mother of oedipus* would have to be construed as a function, e.g. f(o), the phrase *is mother of oedipus* is represented by a predicate.

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Predicates and Logical Operators

We can straightforwardly use **logical operators** in connection with **predicates**. For disambuiguating the scope of logical operators (e.g. negation below) we will use normal parentheses.

- (12) Abraham Lincoln was tall and homely.
- (13) Socrates was a bright man.
- (14) Joe is neither honest nor competent.

TALL(a) \land HOMELY(a) BRIGHT(s) \land MAN(s)

 \neg (HONEST(j) \lor COMPETENT(j))¹⁴ \equiv \neg HONEST(j) $\land \neg$ COMPETENT(j) Section 1: Recap of Lecture 21

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¹⁴Note that we here have to use the inclusive or \lor , rather than the exclusive XOR, since only the former allows for both predicates to be true, which is needed to negate them in a *neither...nor* statement.



Predicates of Predicates

Predicates can also be embedded into other predicates.

- (15) Henry thinks that Anne is beautiful. THINK(h, BEAUTIFUL(a))
- (16) Susan wants to marry Ringo. WANT(s, MARRY(s,r))
- (17) If you are honest, John will perceive you as competent. HONEST(y) \rightarrow PERCEIVE(j, COMPETENT(y))

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Predicates and Quantifiers

Importantly, formulating predicates which involve **quantifications** requires the usage of particular logical operators, since quantifiers require variables, and the variables then need to be further linked to predicates via logical operators.

- (18) All students are diligent. $\forall x(STUDENT(x) \rightarrow DILIGENT(x))$ lit. "For all x it is the case that if x is a student, then x is diligent."
- (19) Some men snore. $\exists x(MAN(x) \land SNORE(x))$ lit. "There exists some x for which it is the case that x is a man and x snores."¹⁵
- (20) No crocodile is warm-blooded. $\neg \exists x (CROCODILE(x) \land WARM-BLOODED(x))$ lit. "It is not the case that there is some x for which x is a crocodile and x is warm-blooded."

¹⁵Note that while the plural *men* suggests that we are talking about 2 or more individuals, the predicate logic formulation is valid for 1 or more individual(s).

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Quantifiers in Natural Language

The set-theoretic definitions of quantifiers (e.g. "some" strictly means there is at least one element in one set shared with another set) are not necessarily adhered to in natural language usage.

Understanding this discrepancy is a subject of current research.

Franke (2014). Typical use of quantifiers. Degen (2015). Processing scalar implicatures.

Van Tiel et al. (2021). Probabilistic pragmatics explains gradience and focality in natural language quantification.



"We conducted a large-scale study on the production of quantity words in English (Experiment [Exp.] 1a). Each of 600 participants described 10 displays showing 432 circles, which were either red or black. To describe these displays, participants freely completed the sentence frame "– of the cir cles are red."" Section 1: Recap of Lecture 21

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Multi-Valent Predicates and Quantifiers

In the case of **multi-valent predicates** being combined with **quantifiers**, we typically have a combination of *variables* and *constant symbols* as arguments of the predicates. Indefinite noun phrases are typically translated using the existential quantifier.

(21) Mary knows all the professors. ∀x(PROFESSOR(x) → KNOW(m,x)) lit. "For all x it is the case that if x is a professor, then Mary knows x."
(22) Susan married a cowboy. ∃x(COWBOY(x) ∧ MARRY(s,x)) lit. "For some x it is the case that x is a cowboy and Susan married x."¹⁶
(23) Ringo lives in a yellow submarine. ∃x(YELLOW(x) ∧ SUBMARINE(x) ∧ LIVE_IN(r,x)) lit. "For some x it is the case that x is yellow, and x is a submarine, and that

lit. "For some x it is the case that x is yellow, and x is a submarine, and that Ringo lives in x."

¹⁶We might be tempted to drop the indefinite determiner and formulate just MARRY(s,c). However, note that "a cowboy" and "the cowboy" are different, in the sense that the former is a not further defined individual in the set of cowboys, while the latter refers to a particular individual.

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Scope Ambiguities

"When a quantifier combines with another quantifier, with negation, or with various other elements [...], it can give rise to **ambiguities of scope**."

Kroeger (2019). Analyzing meaning, p. 72.

(24) Some man loves every woman.

i. $\exists x(MAN(x) \land (\forall y(WOMAN(y) \rightarrow LOVE(x,y))))$ lit. "Fore some x it is the case that x is a man and (for all y it is the case that if y is a woman then x loves y)."

ii. $\forall y(WOMAN(y) \rightarrow (\exists x(MAN(x) \land LOVE(x,y))))$

lit. "For all y it is the case that if y is a woman then there is an x which is a man and loves y."

(25) All that glitters is not gold.

i. $\forall x(GLITTER(x) \rightarrow \neg GOLD(x))$

lit. "For all x it is the case that if x glitters then x is not gold."

ii. $\neg \forall x(GLITTER(x) \rightarrow GOLD(x))$

lit. "It is not the case for all x that if x glitters then x is gold."

Note: In the first case the ambiguity is between whether the existential quantifier scopes over the universal quantifier, or the other way around. In the second example the ambiguity is whether the negation scopes over the universal quantifier or the other way around.

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Section 5: Inference and Predicate Logic





Universal Instantiation

We can now translate the classical types of inferences (which are not Section 1: Recap of Lecture 21 covered by prepositopnal logic) into predicate logic notation. Below is a Section 2: **Historical Notes** classic inference called **universal instantiation**. By using a variable x Section 3: Basic bound by the universal quantifier (Premise 1), and then specifying this Definitions variable as a constant symbol (Premise 2), we adhere to a valid pattern Section 4: Translating to of inference. **Predicate Logic**

(26)	Premise 1: <i>All men are mortal.</i> Premise 2: <i>Socrates is a man.</i>	$\forall x(MAN(x) \rightarrow MORTAL(x))$ MAN(s)	Section 6: Evaluating Predicate Logic Expressions
	Conclusion: Therefore. Socrates		Summary
	is mortal.	MORTAL(s)	References

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Existential Generalization

Another classic example is the so-called **existential generalization**. By asserting that two predicates are true for the same constant symbol (premise 1 and premise 2), we can generalize that there has to be a variable x for which both predicates hold.

(27) Premise 1: Arthur is a lawyer. LAWYER(a) Premise 2: Arthur is honest. HONEST(a)

> Conclusion: Therefore, **some** (= at least one) lawyer is honest. $\exists x(LAWYER(x) \land HONEST(x))$

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Model Theory

"In order to develop and test a set of interpretive rules [...] it is important to provide very explicit descriptions for the test situations. As stated above, this kind of **description of a situation is called a model**, and must include two types of information: (i) **the domain**, i.e., the set of all individual entities in the situation; and (ii) the **denotation sets for the basic vocabulary items** [constant symbols, predicates] in the expressions being analyzed."

Kroeger (2019). Analyzing meaning, p. 240.

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Example Domain

Let us assume our example **domain**, i.e. the so-called **universal set U** is a set of exactly three individuals that we can then use in further more complicated predicate logic expressions.

U = {King Henry VIII, Anne Boleyn, Thomas More}

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Example Denotations

Let us further assume the **denotation sets** of three predicates and three constant symbols. These denotation sets specify which individuals of U a particular expression can possibly denote.

Note: If we give a model world, we could further introduce *situations s* like in Zimmermann & Sternefeld's definitions, and hence further divide the denotation sets according to situations.

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Example Model Evaluation

Based on our **example model**, consisting of the example domain and the example universal set, we can now evaluate the truth values of predicate logic expressions.

- N-place predicates are evaluated by whether the constant symbol, the pair of constant symbols, etc. is a member of the denotation set of the predicate.
- **Logical operators** are evaluated the same way as in propositional logic.
- Quantifiers are evaluated according to subset relations.

See Kroeger (2019). Analyzing meaning, p. 241.

English sentence	logical form	interpretation	truth value
a. Thomas More is a man.	MAN(t)	Thomas More ∈ [[MAN]]	Т
b. Anne Boleyn is a mai or a woman.	n MAN(a) \vee WOMAN(a)	Anne Boleyn $\in (\llbracket MAN \rrbracket \cup \llbracket WOMAN \rrbracket$) T
c. Henry VIII is a man who snores.	$MAN(h) \land SNORE(h)$	Henry VIII $\in (\llbracket MAN \rrbracket \cap \llbracket SNORE \rrbracket)$)	Т
d. All men snore.	$\forall x[MAN(x) \rightarrow SNORE(x)]$	[[MAN]]⊆[[SNORE]]	F
e. No women snore.	$\neg \exists x [WOMAN(x) \land SNORE(x)]$	$] [[WOMAN]] \cap [[SNORE]] = \emptyset$	Т

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Summary

- Predicate logic was developed in the late 19th century as a more "fine-grained" alternative to propositional logic.
- It is built on predicates and functions (constants) which are mathematically defined.
- It allows for universal and existential quantifications.
- It can be evaluated based on a model situation/world.

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Thank You.

Contact:

Faculty of Philosophy General Linguistics Dr. Christian Bentz SFS Wilhelmstraße 19-23, Room 1.24 chris@christianbentz.de Office hours: During term: Wednesdays 10-11am Out of term: arrange via e-mail

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