EBERHARD KARLS
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Faculty of Philosophy
General Linguistics

Syntax & Semantics WiSe 2022/2023

Lecture 20: Propositional Logic

24/01/2023, Christian Bentz

Overview

Semantics Lectures

- **Lecture 18**: Introduction to Semantics
  Kroeger (2019), Chapters 1-2.
- **Lecture 19**: Word Meaning
  Kroeger (2019), Chapter 5-6.
- **Lecture 20**: Propositional Logic
  Kroeger (2019), Chapter 3-4.
  Zimmermann & Sternefeld (2013), Chapter 7.
- **Lecture 21**: Predicate Logic
  Kroeger (2019), Chapter 4.
  Zimmermann & Sternefeld (2013), Chapter 10 (p. 244-258).
- **Lecture 22**: Syntax & Semantics Interface
  Kearns (2011), Chapter 4.
  Zimmermann & Sternefeld (2013), Chapter 4.

Q&As

1. (concerning mock exam ex.1) “Then” is not a pronoun. It is just a more generic time adverb than the complex adverb “yesterday at night”. A pronoun stays in place of a noun, so the pronominalisation test should be flawed by definition because it can only recognise nouns. Could you please provide me a clear definition of the pronominalisation test that explains why words like “then” or “here” would also work as substitutes?

In the lecture, I follow the terminology used in the literature I cite (e.g. Müller, 2019). When in doubt, I refer to the SIL Glossary of Linguistic Terms.[1]

Now, Müller uses the term “pronominalization test”, suggesting that you have to use pronouns for the test. Strictly speaking, however, it would be more correct to use the term “*pro-form* test”, since pronouns are only a subcategory of the more general category of *pro-forms* (see next slide). In this context, forms like *then*, *here*, *now*, are given as pro-forms. Actually, they are explicitely given as *non-examples* of adverbs (see slide below).

[1] https://glossary.sil.org/term

GLOSSARY OF LINGUISTIC TERMS

‹ Back to Bibliographies (Linguistics)

ALL A B C D E F G H I J K L M N O P

Pro-Form

Definition:
A pro-form is a word, substituting for other words, phrases, clauses, or sentences, whose meaning is recoverable from the linguistic or extralinguistic context.

Kinds:

- Interrogative Pro-Form
- Pro-Adjective
- Pro-Adverb
- Pronoun
- Pro-Verb

Examples:
(English)

- Jim cooks better than *she* does.
- He *did* so.

GLOSSARY OF LINGUISTIC TERMS

< Back to Bibliographies (Linguistics)

ALL A B C D E F G H I J K L M N O P

Adverb (Grammar)

Definition:
An adverb is a lexical category whose members have the same syntactic distribution and typically modify adjectives, other adverbs, verbs, or whole clauses or sentences.

Discussion:
The general class *adverb* is a mixture of very different kinds of words, which cover a wide range of semantic concepts and whose syntactic distribution is disparate. The definition of the lexical category *adverb* is language-specific, based on syntactic distribution.

Tip: Any word with lexical content that does not clearly fit the categories *noun, verb,* or *adjective* is usually considered an adverb.

Examples:
(English)

True adverbs in English are words that can be modified by degree words such as the following:

- possibly
- quickly
- well
- far

NonExamples:
(English)

Many words traditionally called adverbs in English are not in the same lexical category as true adverbs because they do not have the same syntactic distribution as true adverbs and cannot be modified by degree words.

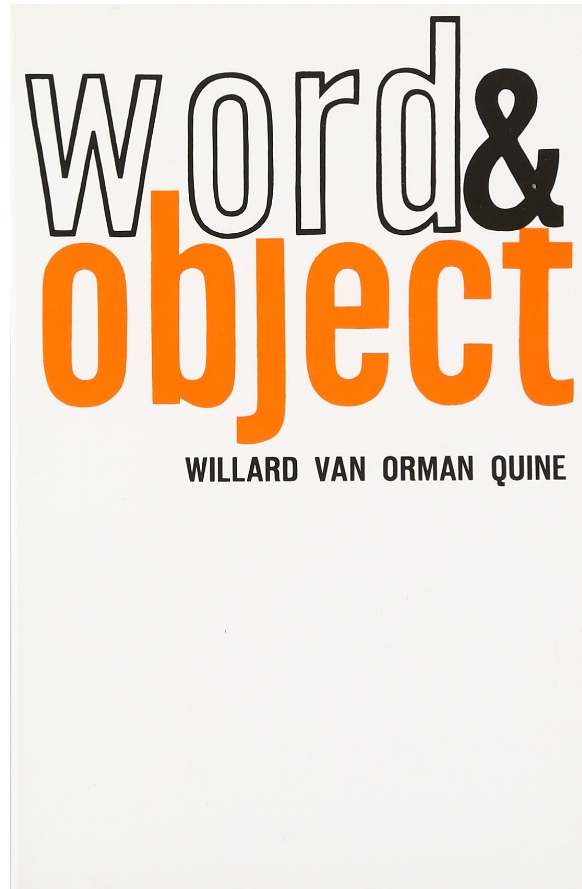
- very
- not
- here
- there
- now
- then
- yesterday

Section 1: Recap of Lecture 20

Meaning as Reference

"What is relevant rather to our purposes is *radical translation*, i.e., translation of the language of a hitherto untouched people [...] The utterances first and most surely translated in such a case are ones keyed to present events that are conspicuous to the linguist and his informant. A rabbit scurries by, the native says 'Gavagai', and the linguist notes down the sentence 'Rabbit' or 'Lo, a rabbit') as tentative translation, subject to testing in further cases."

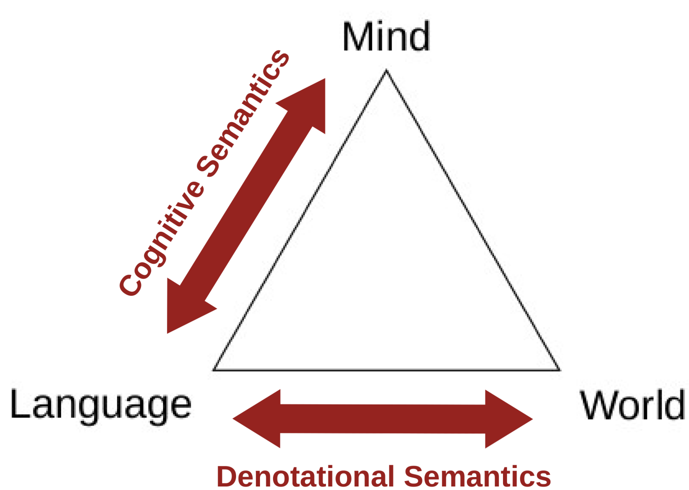
Quine (1960). Word and object, p. 28.



Denotational vs. Cognitive Semantics

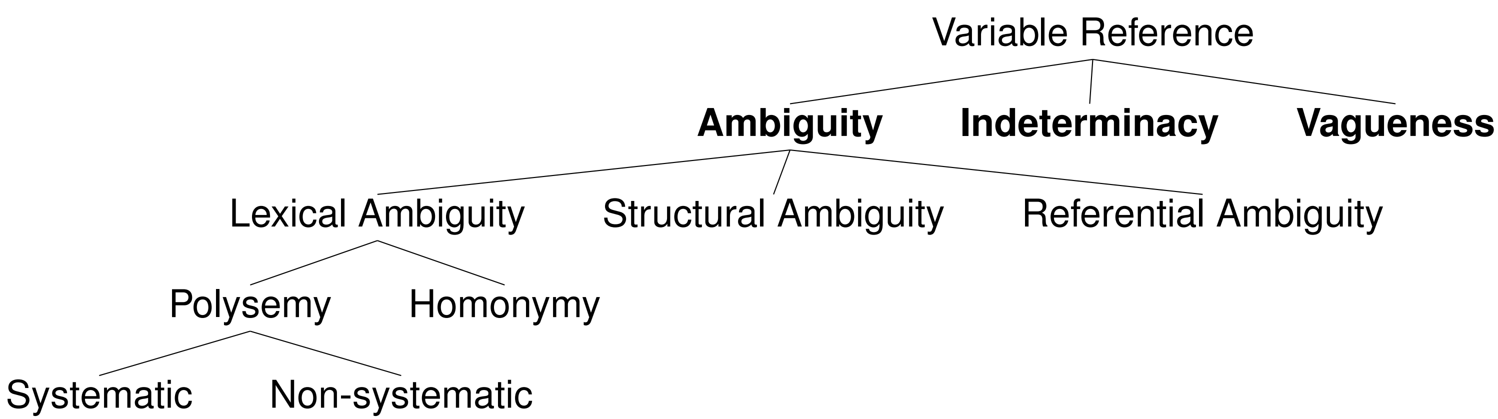
"The basic approach we adopt in this book focuses on the link between linguistic expressions and the world. This approach is often referred to as **denotational semantics** [...] An important alternative approach, **cognitive semantics**, focuses on the link between linguistic expressions and mental representations."

Kroeger (2019). Analyzing meaning, p. 17.



Variable Reference

Even if we assume that reference between forms and meanings is generally possible (i.e. denotational semantics), then there is still the problem of **variable reference**, i.e. *ambiguity*, *indeterminacy* and *vagueness*.



Lexical Ambiguity

"It is possible for a single word to have more than one sense. [...] Words that have two or more senses are said to be **ambiguous** (more precisely, **polysemous** [...])."

Kroeger (2019). Analyzing meaning, p. 23

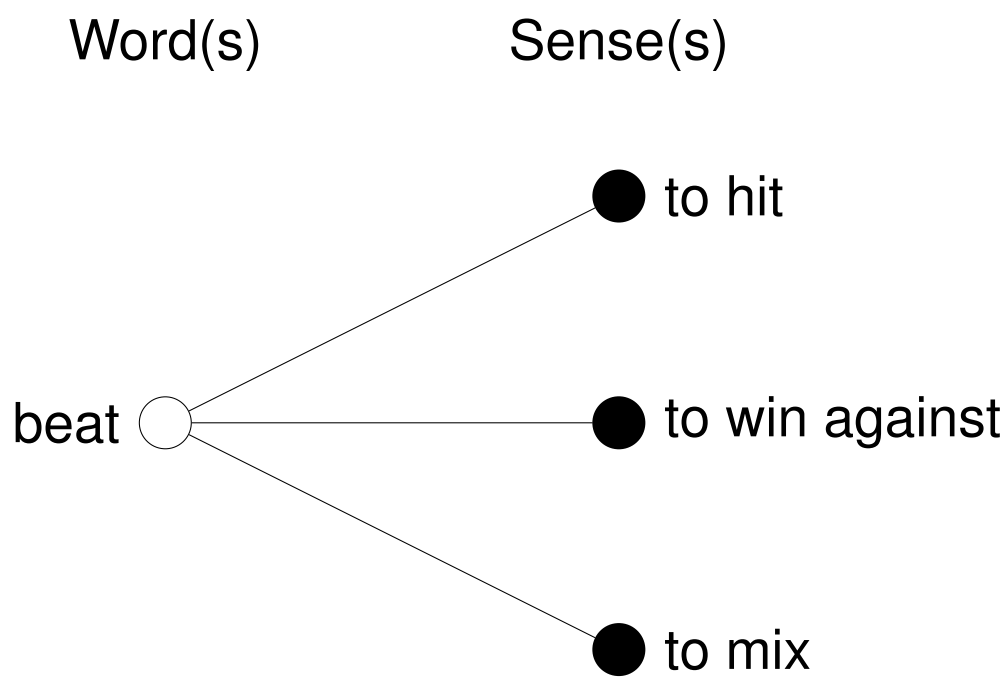
(1) A boiled egg is hard to *beat*.

(2) The farmer allows walkers to cross the field for free, but the bull *charges*.

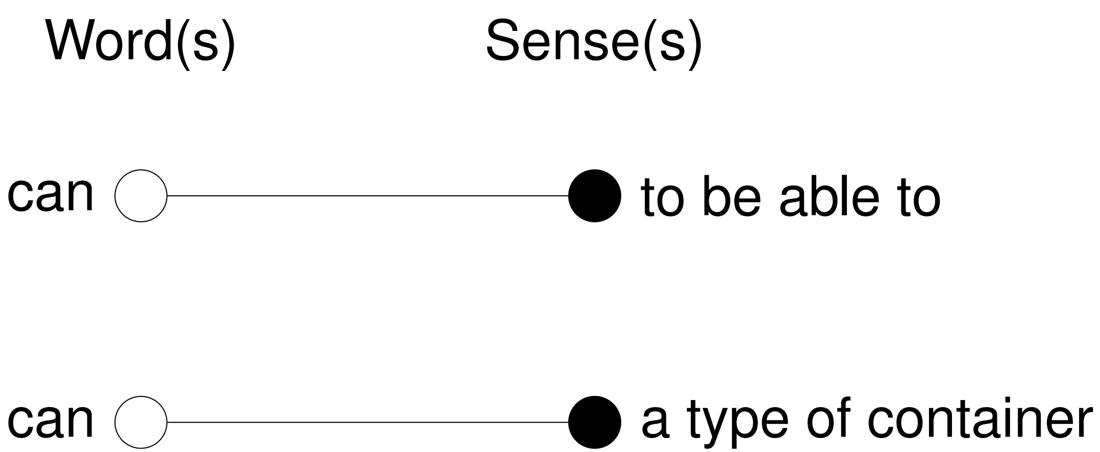
beat, verb
Sense 1: to strike or hit repeatedly
Sense 2: to win against
Sense 3: to mix thoroughly
etc.

https://dictionary.cambridge.org/dictionary/english-german/beat

Ambiguity (Polysemy)



Ambiguity (Homonymy)



Indeterminacy

A type of variable reference, i.e. a word can have variability in its reference despite having a single defined sense. That is, the sense is **indeterminate** with regards to a particular dimension of meaning.

Kroeger (2019). Analyzing meaning, p. 81.

cousin, noun
Sense: a **son or daughter** of one's uncle or aunt.

https://dictionary.cambridge.org/dictionary/english-german/cousin

Note: The term *cousin* in English does not further specify the gender of the person referred to. Hence, it is indeterminate with regards to natural gender. In German, the natural gender is determined by the gender of the article and a suffix (*der Cousin*/ *die Cousin-e*).

Vagueness

A word is **vague** if the "limits of its possible denotations cannot be precisely defined."[2]

Kroeger (2019). Analyzing meaning, p. 81.

tall, adjective
Sense: (of people and thin or narrow objects such as buildings or trees) **higher than normal**

https://dictionary.cambridge.org/dictionary/english-german/tall

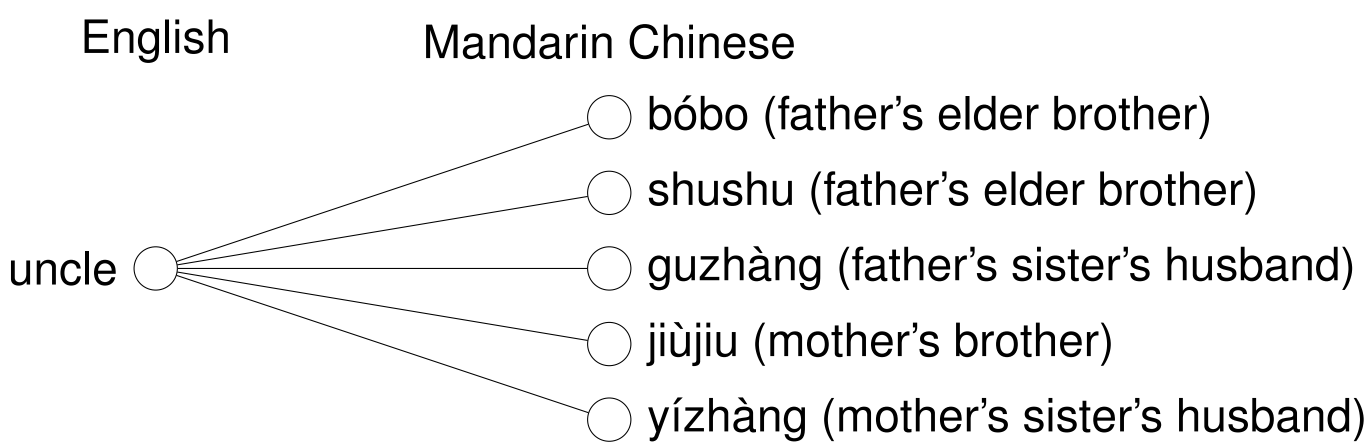
Note: The question here is "what is a *normal* height under which exact conditions?". In fact, this question can be answered precisely by statistics (e.g. more than two standard deviation above average), but humans do not necessarily use such words in a statistically precise way.

[2] Vagueness is sometimes also contrued as a cover term including indeterminacy as a sub-type. However, here the two are argued to be different concepts.

Indeterminacy versus Vagueness

"Another property which may distinguish vagueness from indeterminacy is the degree to which these properties are preserved in translation. Indeterminacy tends to be **language-specific**. There are many interesting and well-known cases where pairs of translation equivalents differ with respect to their degree of specificity."

Kroeger (2019). Analyzing meaning, p. 83.



Ambiguity vs. Vagueness/Indeterminacy

There are a range of tests proposed in the literature which are based on the fact that senses of ambiguous words are **antagonistic**, meaning that they cannot apply simultaneously:

- Zeugma Test
- Identity Test
- Sense Relations Test
- Contradiction Test

Kroeger (2019). Analyzing meaning, p. 84.

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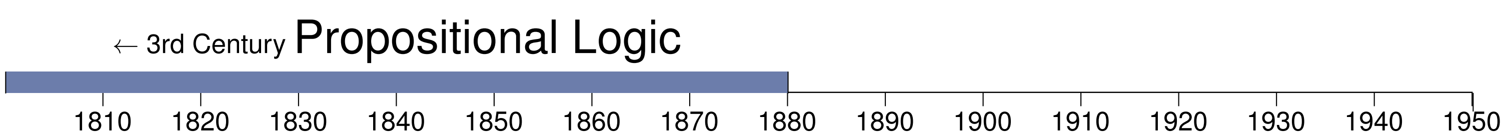
Section 2: Historical Notes

Historical Notes

"In the Hellenistic period, and apparently independent of Aristotle's achievements, the logician Diodorus Cronus and his pupil Philo (see the entry Dialectical school) worked out the beginnings of a logic that took **propositions**, **rather than terms**,[3] as its basic elements. They influenced the second major theorist of logic in antiquity, the **Stoic Chrysippus (mid-3rd c.)**, whose main achievement is the **development of a propositional logic** [...]"

https://plato.stanford.edu/archives/spr2016/entries/logic-ancient/
(accessed 10/02/2021)



[3] A *term* here represents an object, a property, or an action like "Socrates" or "fall", which cannot by itself be true or false. A *proposition* is then a combination of terms which can be assigned a truth value, e.g. "Socrates falls".

Section 3: Basic Concepts

Three levels of meaning

1. **Word meaning**: Meaning assigned to individual words.
2. **Sentence meaning**: Meaning derived via combination of word meanings (compositional).
3. **Utterance meaning** ("speaker" meaning): "The term **utterance meaning** refers to the semantic content plus any pragmatic meaning created by the specific way in which the sentence gets used."

Kroeger (2019). Analyzing meaning, p.5.

Why use Formal Logic?

- We might (to some degree) **overcome** *ambiguity*, *vagueness*, *indeterminacy* inherent to language (if we want to).
- Logic provides precise rules and methods to determine the **relationships between meanings of sentences** (entailments, contradictions, paraphrase, etc.).
- Sytematically testing mismatches between logical inferences and speaker intuitions might help **determining the meanings of sentences**.
- Formal logic helps **modeling compositionality**.
- Formal logic is a **recursive system**, and might hence correctly model recursiveness in language.

Kroeger (2019). Analyzing meaning, p. 54.

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Propositions

Proposition

"The meaning of a simple declarative sentence is called a **proposition**. A proposition is a claim about the world which may (in general) be true in some situations and false in others."

Kroeger (2019), p. 35.

"To know the meaning of a [declarative] sentence is to know what the world would have to be like for the sentence to be true."

Kroeger (2019), p. 35, citing Dowty et al. (1981: 4).

(3) *Mary snores.*
(4) *King Henry VIII snores.*
(5) *The unicorn in the garden snores.*

Formal Definition: Extension

Remember that in **denotational semantics** meaning is construed as the mapping between a given word and the real-world object it refers to (reference theory of meaning). More generally, words, phrases or sentences are said to have **extensions**, i.e. real-world situations they refer to.

Zimmermann & Sternefeld (2013), p. 71.

Type of expression	**Type of extension**	**Example**	**Extension of example**
proper name	individual	*Paul*	Paul McCartney
definite description	individual	*the biggest German city*	Berlin
noun	set of individuals	*table*	the set of tables
intransitive verb	set of individuals	*sleep*	the set of sleepers
transitive verb	set of pairs of individuals	*eat*	the set of pairs $\langle \textit{eater}, \textit{eaten} \rangle$
ditransitive verbs	set of triples of individuals	*give*	the set of triples $\langle \textit{donator}, \textit{recipient}, \textit{donation} \rangle$

Formal Definition: Extensions

"Let us denote the **extension** of an expression *A* by putting double brackets '$[\![\,]\!]$' around *A*, as is standard in semantics. The extension of an expression depends on the **situation *s*** talked about when uttering *A* ; so we add the index *s* to the closing bracket."

Zimmermann & Sternefeld (2013), p. 85.

$[\![\text{Paul}]\!]_s$ = Paul McCartney[4]
$[\![\text{the biggest German city}]\!]_s$ = Berlin
$[\![\text{table}]\!]_s = \{\text{table}_1, \text{table}_2, \text{table}_3, \ldots, \text{table}_n\}$[5]
$[\![\text{sleep}]\!]_s = \{\text{sleeper}_1, \text{sleeper}_2, \text{sleeper}_3, \ldots, \text{sleeper}_n\}$
$[\![\text{eat}]\!]_s = \{\langle\text{eater}_1, \text{eaten}_1\rangle, \langle\text{eater}_2, \text{eaten}_2\rangle, \ldots, \langle\text{eater}_n, \text{eaten}_n\rangle\}$

[4] Zimmermann & Sternefeld just put the full proper name in brackets here, Kroeger follows another convention and just puts the first letter in lower case, e.g. $[\![\text{p}]\!]_s$.

[5] Kroeger (2019) uses upper case notation for both nouns and predicates, e.g. TABLE and SLEEP respectively.

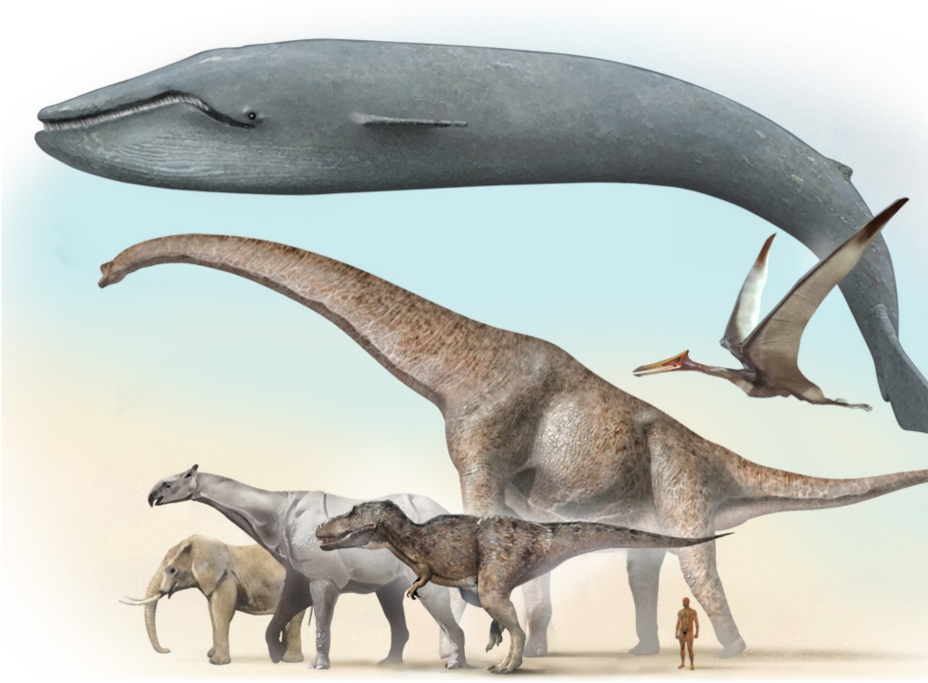
Formal Definition: Frege's Generalization

"The **extension of a sentence S** is its **truth value**, i.e., 1 if S is true and 0 if S is false."

Zimmermann & Sternefeld (2013), p. 74.

S_1: The African elephant is the biggest land mamal.
$[\![S_1]\!]_s = 1$, with s being 21st century planet earth.
$[\![S_1]\!]_s = 0$, with s being planet earth.

S_2: The African elephant is the biggest mamal.
$[\![S_2]\!]_s = 0$, with s being 21st century planet earth.
$[\![S_2]\!]_s = 0$, with s being planet earth.



Formal Definition: Proposition

"The **proposition expressed by a sentence** is the **set of possible cases [situations]** of which that sentence is true."
Zimmermann & Sternefeld (2013), p. 141.

Coin-flip example:

situation	flip1	flip2
1	heads	heads
2	tails	tails
3	heads	tails
4	tails	heads

Sentence	**Proposition**
S_1: only one flip landed heads up	$[\![S_1]\!] = \{3, 4\}$
S_2: all flips landed heads up	$[\![S_2]\!] = \{1\}$
S_3: flips landed at least once tails up	$[\![S_3]\!] = \{2, 3, 4\}$
etc.	etc.

Formal Definition: Proposition

We thus have the following definitions:

- The **proposition** expressed by a sentence is the set of possible situations of which that sentence is true.
- A sentence S is true of a possible situation s if and only if $[\![S]\!]_s = 1$.
- $[\![S]\!]$, in turn, is then the proposition expressed by S, such that: $[\![S]\!] \equiv \{s : [\![S]\!]_s = 1\}$
- A sentence S is true of a possible situation s if and only if $s \in [\![S]\!]$, formally: $[\![S]\!]_s = 1$ iff $s \in [\![S]\!]$.

Adopted from Zimmermann & Sternefeld (2013), p. 144.

Note: Zimmermann & Sternefeld extent the definition from situations s to **possible worlds** w in order to capture the totality of all possible cases rather than cases specific to a situation.

Types of Sentences and Propositions

- **Analytic sentence (Tautology)**: A sentence which is true in every situation, i.e. the proposition is a set which includes all possible situations.
  Example: *Today is the first day of the rest of your life*.
- **Contradiction**: A sentence which is false in every situation, i.e. the proposition is an empty set.
  Example: *Your children are not your children*.[6]
- **Synthetic sentence**: A sentence which is either true or false depending on the situation, i.e. the proposition is an non-empty subset of all possible situations.
  Example: *The African elephant is the biggest land mamal*.

[6] There are potentially situations in which this sentence might be true, depending on the different senses *child* might have.

Inference

Inference

"[...] knowing that one fact or set of facts is true gives us an adequate basis for concluding that some other fact is also true. **Logic** is the **science of inference**."

Premisses: The facts which form the basis of the inference.
Conclusions: The fact which is inferred.

Kroeger (2019). Analyzing meaning, p. 55.

(6) Premise 1: *Either Joe is crazy or he is lying.*
Premise 2: *Joe is not crazy.*

Conclusion: *Therefore, Joe is lying*.

Syllogism

"An important variety of deductive argument in which a conclusion follows **from two or more premises**; especially the categorical syllogism."

http://www.philosophypages.com/dy/s9.htm#syl

Categorical Syllogism

"A logical argument consisting of **exactly three categorical propositions, two premises and the conclusion,** with a total of exactly three categorical terms, each used in only two of the propositions."

http://www.philosophypages.com/dy/c.htm#casyl

Note: The distinction between *syllogism* and *categorical syllogism* is typically dropped by logicians, and inferences drawn from premises are called syllogisms in general.

Types of Inference

There are (at least) **three types of inferences** that are relevant for analyzing sentence meanings:

- Inferences based on **content words**
- Inferences based on **logical words** (rather than content words)
- Inferences based on **quantifiers** (and logical words)

Kroeger (2019). Analyzing meaning, p. 56.

Content Word Inference

If inferences are drawn based purely on **content words**, then we are strictly speaking outside the domain of logic, since logic deals with generalizable patterns of inference, rather than ideosyncrasies of individual words and their meanings.

(7) Premise: *John **killed** the wasp.*

Conclusion: *Therefore, the wasp **died***.

Note: The validity of the inference here depends on our understanding and definition of the words *killed* and *died*. *Kill* is typically defined as "to cause sb. or sth. to die". Hence, the inference is valid.

Logical Word Inference

If inferences are drawn based purely on the **meaning of logical words** (operators), then the inference is generalizable to a potentially infinite number of premisses and conclusions. Note that we can replace the propositions by placeholders. Here, we are in the domain of **propositional logic**.

(8) Premise 1: ***Either*** *Joe is crazy* ***or*** *he is lying.*
Premise 2: *Joe is* ***not*** *crazy.*

Conclusion: ***Therefore***, *Joe is lying*.

(9) Premise 1: ***Either*** *x* ***or*** *y.*
Premise 2: ***not*** *x.*

Conclusion: ***Therefore***, *y*.

Quantifier Inference

If quantifiers are used (on top of other logical operators), pure propositional logic is not sufficient anymore. We are then in the domain of **predicate logic**.

(10) Premise 1: ***All*** *men are mortal.*
Premise 2: *Socrates is a man.*

Conclusion: *Therefore, Socrates is mortal.*

Section 4: Propositional Logic

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Propositional Operators

We will here use the following operators:

Operator	**Alternative Symbols**	**Name**	**English Translation**
$\neg$	$\sim$, !	negation	*not*
$\wedge$	., &	conjunction	*and*
$\vee$	+, ||	disjunction (inclusive *or*)	*or*
XOR	EOR, EXOR, $\oplus$, $\veebar$	exclusive *or*	*either ... or*
$\rightarrow$	$\Rightarrow$, $\supset$	material implication[7]	*if ..., then*
$\leftrightarrow$	$\Leftrightarrow$, $\equiv$	material equivalence[8]	*if, and only if ..., then*

Note: We will here assume that the English translations and the operators themselves are indeed equivalent in their meanings. However, in language usage, this might not actually be the case.

[7] aka *conditional*
[8] aka *biconditional*

Truth Tables

In a **truth table** we identify the extensions of (declarative) sentences as truth values. In the notation typically used, the variables *p* and *q* represent such **truth values of sentences**.[9] The left table below gives the notation according to Zimmermann & Sternefeld, the right table according to Kroeger. We will use the latter for simplicity.

$[\![S_1]\!]_s$	$[\![S_2]\!]_s$	$[\![S_1]\!]_s \wedge [\![S_2]\!]_s$
1	1	1
1	0	0
0	1	0
0	0	0

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

[9] Kroeger (2019), p. 58 writes that *p* and *q* are variables that represent propositions. However, according to the definitions we have given above this is strictly speaking not correct.

Negation

"When we have said that p and ¬p must have opposite truth values in any possible situation, we have provided a definition of the negation operator; nothing needs to be known about the specific meaning of p."
Kroeger (2019). Analyzing meaning, p. 59.

p	¬p
T	F
F	T

(11) S_1: *Peter is your child.*
$p \equiv [\![S_1]\!]_s \in \{T, F\}$
$\neg p \equiv \neg[\![S_1]\!]_s \in \{T, F\}$
Example: if the situation *s* is such that Peter is *not* the child of the person referred to as *you*, then $p \equiv [\![S_1]\!]_s = F$, and $\neg p \equiv \neg[\![S_1]\!]_s = T$, otherwise the other way around.

Conjunction

"In the same way, the operator ∧ 'and' can be defined by the truth table [below]. This table says that p∧q (which is also sometimes written p&q) is true just in case both p and q are true, and false in all other situations."

Kroeger (2019). Analyzing meaning, p. 59.

p	q	p∧q
T	T	T
T	F	F
F	T	F
F	F	F

(12) S_1: *Peter is your child.*
$p \equiv [\![S_1]\!]_s \in \{T, F\}$

(13) S_2: *The moon is blue.*
$q \equiv [\![S_2]\!]_s \in \{T, F\}$

$$p \wedge q \equiv [\![S_1]\!]_s \wedge [\![S_2]\!]_s \in \{T, F\}$$

Example: if the situation *s* is such that Peter *is* the child of the person referred to as *you*, but the moon is *not* blue, then $p \wedge q \equiv [\![S_1]\!]_s \wedge [\![S_2]\!]_s = F$.

Disjunction (Inclusive *or*)

"The operator ∨ 'or' is defined by the truth table [below]. This table says that p∨q is true whenever either p is true or q is true; it is only false when both p and q are false. Notice that this *or* of standard logic is the *inclusive or*, corresponding to the English phrase *and/or*, because it includes the case where both p and q are true."

Kroeger (2019). Analyzing meaning, p. 60.

p	q	p∨q
T	T	T
T	F	T
F	T	T
F	F	F

(14) S_1: *Peter is your child.*
$\text{p} \equiv [\![S_1]\!]_s \in \{T, F\}$

(15) S_2: *The moon is blue.*
$\text{q} \equiv [\![S_2]\!]_s \in \{T, F\}$

$$\text{p} \vee \text{q} \equiv [\![S_1]\!]_s \vee [\![S_2]\!]_s \in \{T, F\}$$

Example: if the situation *s* is such that Peter *is not* the child of the person referred to as *you*, but the moon *is* indeed blue, then $\text{p} \vee \text{q} \equiv [\![S_1]\!]_s \vee [\![S_2]\!]_s = \text{T}$.

Exclusive *or*

"[The table below] shows how we would define this exclusive "sense" of *or*, abbreviated here as XOR. The table says that p XOR q will be true whenever either p or q is true, but not both; it is false whenever p and q have the same truth value."

Kroeger (2019). Analyzing meaning, p. 60.

p	q	p XOR q
T	T	F
T	F	T
F	T	T
F	F	F

(16) S_1: *Peter is your child.*
$\text{p} \equiv [\![S_1]\!]_s \in \{T, F\}$

(17) S_2: *The moon is blue.*
$\text{q} \equiv [\![S_2]\!]_s \in \{T, F\}$

$$\text{p XOR q} \equiv [\![S_1]\!]_s \text{ XOR } [\![S_2]\!]_s \in \{T, F\}$$

Example: if the situation *s* is such that Peter *is* the child of the person referred to as *you*, and the moon *is* indeed blue, then
$\text{p XOR q} \equiv [\![S_1]\!]_s \text{ XOR } [\![S_2]\!]_s = \text{F}$.

Material Implication (Conditional)

"The material implication operator $\rightarrow$ is defined by the truth table [below]. (The formula p$\rightarrow$q can be read as *if p (then) q*, *p only if q*, or *q if p*.) The truth table says that p$\rightarrow$q is defined to be false just in case p is true but q is false; it is true in all other situations."
Note: p is called the *antecedent* here, and q the *consequent*.

Kroeger (2019). Analyzing meaning, p. 61.

p	q	p $\rightarrow$ q
T	T	T
T	F	F
F	T	T
F	F	T

(18) S_1: *Peter is your child.*
$p \equiv [\![S_1]\!]_s \in \{T, F\}$

(19) S_2: *The moon is blue.*
$q \equiv [\![S_2]\!]_s \in \{T, F\}$

$p \rightarrow q \equiv [\![S_1]\!]_s \rightarrow [\![S_2]\!]_s \in \{T, F\}$

Example: if the situation *s* is such that Peter *is* the child of the person referred to as *you*, but the moon *is not* blue, then $p \rightarrow q \equiv [\![S_1]\!]_s \rightarrow [\![S_2]\!]_s$ = F. In all other situations, it is T.

Material Equivalence (Biconditional)

"The formula p↔q (read as p *if and only if* q) is a short-hand or abbreviation for: (p→q) ∧ (q→p). The **biconditional** operator is defined by the truth table [below]."

Kroeger (2019). Analyzing meaning, p. 61.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(20) S_1: *Peter is your child.*
$p \equiv [\![S_1]\!]_s \in \{T, F\}$

(21) S_2: *The moon is blue.*
$p \equiv [\![S_2]\!]_s \in \{T, F\}$

$$p \leftrightarrow q \equiv ([\![S_1]\!]_s \leftrightarrow [\![S_2]\!]_s) \in \{T, F\}$$

Example: if the situation *s* is such that Peter *is* the child of the person referred to as *you*, and the moon *is* blue, or if both is *not* the case, then $p \leftrightarrow q \equiv [\![S_1]\!]_s \leftrightarrow [\![S_2]\!]_s$ = T. In all other situations, it is F.

Building Truth Tables for Complex Sentences

We will follow the following four steps to analyze the sentence below:

1. Identify the **logical words** and translate them into **logical operators**
2. **Decompose the sentence** into its component declarative parts and assign **variables** to them (i.e. p and q).
3. Translate the whole sentence into **propositional logic notation**
4. Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

Example Sentence: *If the president is either crazy or he is lying, and it turns out he is lying, then he is not crazy.*

First Step

Identify the **logical words** and translate them into **logical operators**.

If the president is **either** crazy **or** he is lying, **and** it turns out he is lying, **then** he is **not** crazy.

- if ... then: $\rightarrow$ (material implication)
- either ... or: XOR (exclusive *or*)
- and: $\wedge$ (conjunction)
- not: $\neg$ (negation)

Second Step

Decompose the sentence into its component declarative parts and assign **variables** to them (i.e. p and q).

If **the president is** either **crazy** or he **is lying**, and it turns out he is lying, then he is not crazy.

- p: the president is crazy
- q: the president is lying

Note: We make the assumption here that the pronoun *he* refers back to the NP introduced earlier in the discourse, i.e. *the president*.

Third Step

Translate the whole sentence into **propositional logic notation**.

If the president is either crazy or he is lying, and it turns out he is lying, then he is not crazy.

- p: the president is crazy
- $\neg$p: the president is not crazy
- q: the president is lying
- p XOR q: the president is either crazy or he is lying
- $\wedge$ q: and the president is lying
- $\rightarrow$: if the president ... then the president ...

Note: We have to break statements down to simple declarative sentences by ignoring such formulations as *it turns out*. We also have to understand that the XOR and $\wedge$ statements are “embedded” in the $\rightarrow$ statement.

Overall result: ((p XOR q) $\wedge$ q) $\rightarrow$ $\neg$p

Fourth Step

Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

$$((\mathbf{p} \text{ XOR } \mathbf{q}) \wedge q) \rightarrow \neg p$$

p	q
T	T
T	F
F	T
F	F

Fourth Step

Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

$$((\mathbf{p\ XOR\ q}) \wedge q) \rightarrow \neg p$$

p	q	p XOR q
T	T	F
T	F	T
F	T	T
F	F	F

Fourth Step

Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

$$((\mathbf{p\ XOR\ q}) \land \mathbf{q}) \to \neg p$$

p	q	p XOR q	(p XOR q) $\land$ q
T	T	F	F
T	F	T	F
F	T	T	T
F	F	F	F

Fourth Step

Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

$$((p \text{ XOR } q) \wedge q) \rightarrow \neg \mathbf{p}$$

p	q	p XOR q	(p XOR q) ∧ q	¬ p
T	T	F	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	F	T

Fourth Step

Start the truth table with the variables (i.e. p and q) **to the left**, and then add operators step by step (from the most embedded to the outer layers).

$$\mathbf{((p\ XOR\ q) \wedge q) \rightarrow \neg p}$$

p	q	p XOR q	(p XOR q) $\wedge$ q	$\neg$ p	((p XOR q) $\wedge$ q) $\rightarrow$ $\neg$p
T	T	F	F	F	**T**
T	F	T	F	F	**T**
F	T	T	T	T	**T**
F	F	F	F	T	**T**

Section 5: Beyond Propositional Logic

Beyond Propositional Logic

"The propositional logic outlined in this section is an important part of the logical metalanguage for semantic analysis, but it is not sufficient on its own because it is concerned only with **truth values** [of whole sentences]. We need a way to go beyond p and q, to represent the actual meanings of **the basic propositions** we are dealing with."
Kroeger (2019). Analyzing meaning, p. 66.

Example Sentences (Set 1):

p: John is hungry.
q: John is smart.
r: John is my brother.

Example Sentences (Set 2):

p: John snores.
q: Mary sees John.
r: Mary gives George a cake.

Note: Propositional logic assigns variables (p, q, r) to whole declarative sentences, and hence is "blind" to the fact that the first set of sentences shares both the same subject, and the copula construction, whereas the second set of sentences uses predicates of different valencies and different subjects and objects.

Beyond Propositional Logic

A second major limitation of propositional logic is that it cannot take into account **quantifications**, and hence cannot decide on the truth values of the classical syllogisms below.

(22) Premise 1: ***All*** *men are mortal.*
Premise 2: *Socrates is a man.*

Conclusion: *Therefore, Socrates is mortal.*

(23) Premise 1: *Arthur is a lawyer.*
Premise 2: *Arthur is honest.*

Conclusion: *Therefore,* ***some (= at least one)*** *lawyer is honest.*

Summary

Summary

- Formal logic is a tool to capture the **compositionality of meaning at the sentence level**.
- There are different **types of formal logical languages**, e.g. propositional logic and predicate logic.
- **Propositional logic** is able to deal with inferences based on **logical words** (and, or, not, etc.).
- It is limited by not including more fine-grained **predicate-level distinctions** and **quantifiers**.

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Thank You.

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