



# Semantics & Pragmatics SoSe 2022

## Lecture 6: Formal Semantics III (Second-Order Logic)



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# Overview

Section 1: Recap of Lecture 5

Section 2: Historical Notes

Section 3: Beyond First-Order Logic

Section 3: The Vocabulary

Shared with First-Order Logic

Special to Second-Order Logic

Translation Key

Section 4: The Syntax of Second-Order Logic

Recursive Definition

Examples of Valid and Invalid Formulas

Section 5: The Semantics of Second-Order Logic

Summary

References



# Feedback



Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



---

## **Section 1: Recap of Lecture 5**



# The Vocabulary

Similar as for propositional logic, we can define a **language  $L$  for predicate logic**. In this case, the “vocabulary” of  $L$  consists of

- ▶ a (potentially infinite) supply of **constant symbols** (e.g.  $a, b, c$ , etc.),
- ▶ a (potentially infinite) supply of **variable symbols** representing the constants (e.g.  $x, y, z$ , etc.),
- ▶ a (potentially infinite) supply of **predicate symbols** (e.g.  $A, B, C$ , etc.),
- ▶ the **connectives** (e.g.  $\neg, \wedge, \vee, \rightarrow$ , etc.),
- ▶ the **quantifiers**  $\forall$  and  $\exists$ ,
- ▶ as well as the round brackets ‘(’ and ‘)’.
- ▶ (The equal sign ‘=’.)

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



## English sentences:

- (1) Socrates admires someone.
- (2) Socrates is admired by someone.
- (3) All teachers are friendly.
- (4) Some teachers are friendly.
- (5) Some friendly people are teachers.
- (6) All teachers are unfriendly.
- (7) Some teachers are unfriendly.

## Translations:

- (1)  $\exists y A s y$
- (2)  $\exists x A x s$
- (3)  $\forall x (T x \rightarrow F x)$
- (4)  $\exists x (T x \wedge F x)$
- (5)  $\exists x (F x \wedge T x)$
- (6)  $\forall x (T x \rightarrow \neg F x)$
- (7)  $\exists x (T x \wedge \neg F x)$

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References

## Notes:

We have to add  $Tx$ : *x is a teacher* to the key.

Due to the so-called commutativity of  $\wedge$ , i.e.  $\phi \wedge \psi \equiv \psi \wedge \phi$ , we have that the predicate logic expressions in (4) and (5) are seen as equivalent too. However, the asymmetry might be seen as actually relevant in the natural language examples.

Generally, we have that  $\forall x \neg \phi \equiv \neg \exists x \phi$ .



# The Syntax: Recursive Definition

Given the vocabulary of  $L$  we then define the following clauses to create formulas of  $L$ .

- (i) If  $A$  is an  $n$ -ary predicate letter in the vocabulary of  $L$ , and each of  $t_1, \dots, t_n$  is a constant or a variable in the vocabulary of  $L$ , then  $At_1, \dots, t_n$  is a formula in  $L$ .
- (ii) If  $\phi$  is a formula in  $L$ , then  $\neg\phi$  is too.
- (iii) If  $\phi$  and  $\psi$  are formulas in  $L$ , then  $(\phi \wedge \psi)$ ,  $(\phi \vee \psi)$ ,  $(\phi \rightarrow \psi)$ , and  $(\phi \leftrightarrow \psi)$  are too.
- (iv) If  $\phi$  is a formula in  $L$  and  $x$  is a variable, then  $\forall x\phi$  and  $\exists x\phi$  are formulas in  $L$ .
- (v) Only that which can be generated by the clauses (i)-(iv) in a finite number of steps is a formula in  $L$ .

Gamut, L.T.F (1991). Volume 1, p. 75.

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



## Clarification: the number $n$

“In general,  $n$ -ary predicates may be introduced for any whole number  $n$  larger than zero.”

Gamut (1991), Volume 1, p. 67.

“More precisely, a lexicon of predicate logic is a function assigning to any natural number  $n \geq 0$  a set  $\text{PRED}_{n,L}$  of (primitive) symbols [...] For  $n = 0$ ,  $\text{PRED}_n$  contains the individual constants of  $L$ ; and if  $n \geq 1$ , the members of  $\text{PRED}_n$  are called  $n$ -place predicates (of  $L$ ).”

Zimmermann and Sternefeld (2013), p. 245.

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References





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Zimmermann and Sternefeld (2013), p. 245.

**Predicates:**  $n > 0$

**Functions:**  $n \geq 0$

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



## Examples of **Valid** and **Invalid** Formulas

Formula	Rule Applied
$Aa$ ✓	(i)
$Ax$ ✓	(i)
$Aab$ ✓	(i)
$Axy$ ✓	(i)
$\neg Axy$ ✓	(i) and (ii)
$Aa \rightarrow Axy$ ✓	(i) and (iii)
$\forall x(Aa \rightarrow Axy)$ ✓	(i), (iii), and (iv)
$\forall xAa \rightarrow Axy$ ✓	(i), (iii), and (iv)
$a$ ✗	—
$A$ ✗	—
$\forall$ ✗	—
$\forall(Axy)$ ✗	—

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



## Definition: *Formula vs. Sentence*

There is a further distinction between **formulas** and **sentences** in predicate logic. Namely, sentences are a subset of formulas for which it holds that: “A sentence is a formula in  $L$  which **lacks free variables**.”<sup>1</sup>

Gamut, L.T.F (1991). Volume 1, p. 77.

### Sentence

$Aa$

$\forall x(Fx)$

$\forall x(Ax \rightarrow \exists yBy)$

### Not a Sentence (but Formula)

$Ax$

$Fx$

$Ax \rightarrow \exists yBy$

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References

<sup>1</sup>Free variables, in turn, are precisely defined by Gamut (1991), p.77 in their Definition 3. We will simply state here that a variable is free if it is not within the scope of a quantifier.



## Model Theory

“In order to develop and test a set of interpretive rules [...] it is important to provide very explicit descriptions for the test situations. As stated above, this kind of **description of a situation is called a model**, and must include two types of information: (i) **the domain**, i.e., the set of all individual entities in the situation; and (ii) the **denotation sets for the basic vocabulary items** [constant symbols, predicates] in the expressions being analyzed.”

Kroeger (2019). *Analyzing meaning*, p. 240.





# Interpretation Functions

“The interpretation of the constants in  $L$  will therefore be an attribution of some entity in  $D$  to each of them, that is, a function with the set of constants in  $L$  as its domain and  $D$  as its range. Such functions are called **interpretation functions**.”

$$I(c) = e. \quad (1)$$

“ $I(c)$  is called the *interpretation of a constant  $c$* , or its *reference* or its *denotation*, and if  $e$  is the entity in  $D$  such that  $I(c) = e$ , then  $c$  is said to be one of  $e$ ’s names ( $e$  may have several different names).”

Gamut, L.T.F (1991). Volume 1, p. 88.

## Example

$$I = \{ \langle m, e_1 \rangle, \langle s, e_1 \rangle, \langle v, e_1 \rangle \}$$

$$I(m) = e_1$$

$$I(s) = e_1$$

$$I(v) = e_1$$

Translation key: m: morning star; s: evening star; v: venus.

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



## Valuation Example

Given a Model of the world  $\mathbf{M}$ , consisting of  $D$  and  $I$ , and some formula  $\phi$  which adheres to predicate logic syntax (and which consists of atomic sentences and or quantifications with bound variables), we can then evaluate the truth of  $\phi$  as follows.

### Model $\mathbf{M}$

$$D = \{e_1, e_2, e_3\}$$

$$I = \{\langle j, e_1 \rangle, \langle p, e_2 \rangle, \langle m, e_3 \rangle, \langle S, \{\langle I(j), I(m) \rangle, \langle I(p), I(m) \rangle\} \rangle\}$$

Translation key: j: John; p: Peter; m: morning star; Sxy: x sees y.

### Valuation

“John sees the morning star”:  $V_M(Sjm) = 1$  (according to (i))

“Everybody sees the morning star”:  $V_M(\forall x Sxm) = 0$  (according to (vii))<sup>2</sup>

<sup>2</sup>This valuation gives 0 since the morning star (m) is a constant c in L, but it does not see itself, i.e.  $\langle I(m), I(m) \rangle \notin S$ .

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



---

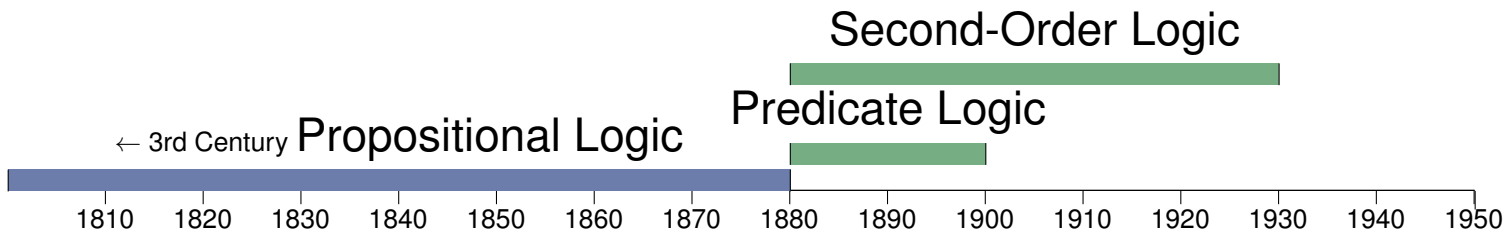
## **Section 2: Historical Notes**



## Historical Background

“Second-order logic was introduced by Frege in his *Begriffsschrift* (1879) who also coined the term “second order” (“zweiter Ordnung”) in (1884: paragraph 53). It was widely used in logic until the 1930s, when set theory started to take over as a foundation of mathematics.”

<https://plato.stanford.edu/entries/logic-higher-order/>



Section 1: Recap of Lecture 5

Section 2: Historical Notes

Section 3: Beyond First-Order Logic

Section 3: The Vocabulary

Section 4: The Syntax of Second-Order Logic

Section 5: The Semantics of Second-Order Logic

Summary

References





## Historical Background

“In present terminology, the thesis is that languages with **variables ranging over properties and relations** are natural extensions of languages with **variables ranging over objects.**”

Shapiro (1991), p. 203.

$x, y, z$ : variables ranging over “objects” (i.e. constants)

$X, Y, Z$ : variables ranging over “properties” and relations (i.e. predicates)

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



## Dispute over Second-Order Logic

It is disputed whether second-order predicates are necessarily needed in logical systems generally, and for natural language logic in particular. Some of the reasons for this include:

- ▶ There is **no completeness theorem** for second-order logic (see Gamut 1991, Volume 1, p. 171), while for first-order logic there is.
- ▶ W. V. Quine rejected the idea that **quantification over predicates** makes sense. He conceptualized predicates as an abbreviation for an incomplete sentence, e.g.  $F$  standing for “...is friendly”, and such *incomplete sentences* are not to be seen as *objects* to quantify over.

See also discussion on [https://en.wikipedia.org/wiki/Second-order\\_logic](https://en.wikipedia.org/wiki/Second-order_logic) under *History and disputed value*.

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



---

## Section 3: Beyond First-Order Logic



## Beyond First-Order Logic

We have seen that predicate logic is an extension of propositional logic, by introducing predicates and quantifiers. **First-Order Predicate Logic** might itself be superseded by another logical system, called **Second-Order Predicate Logic**.

Gamut, L.T.F (1991). Volume 1, p. 168.

Take the following English sentences:

- (1) Mars is red.
- (2) Red is a color.
- (3) Mars has a color.
- (4) John has at least one thing in common with Peter.

How can we translate these into logical expressions?

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



The adjective “red” is a **property of individuals**. Hence, the first sentence can be straightforwardly translated into predicate logic notation as

(5)  $Rm$  ( **$Rx$** :  $x$  is red,  $m$ : Mars)

What about the second sentence? We could stick with standard predicate notation and translate it into

(6)  $Cr$  ( $Cx$ :  $x$  is a color,  **$r$** : red)

Note however, that now we have treated “red” once as a property of individuals in (5), and once as an individual itself in (6). In predicate logic terms it is once represented as a **predicate constant**, and once as a **constant**.

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



## Second-Order Predicates

To circumvent this discrepancy, we can construe the predicate *x is a color* not as a property, but as a **property of properties**.  $\mathcal{C}$  then represents a so-called **second-order property**, i.e. a **second-order predicate** over the first-order predicate *x is red*.

Instead of

(7)  $\mathcal{C}r$  ( $\mathcal{C}x$ : *x is a color*,  $r$ : *red*),

we then get

(8)  $\mathcal{C}R$  ( $\mathcal{C}X$ : *X is a predicate with the property of being a color*,  $Rx$ : *x is red*)

Note: We introduce **two** new sets of symbols here compared to standard predicate logic, a) the set of *second-order predicates*, and b) the set of *first-order predicate variables*. See details below.

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



## First-Order and Second-Order Logic

A **second-order logic** language  $L'$  is then an extension to a standard predicate logic language  $L$  by adding second-order predicates to  $L$ . The original language  $L$  is then sometimes referred to as **first-order logic** language.

### Further Examples:

- (9)  $\exists X(CX \wedge Xm)$  (English sentence: “Mars has a color.”)
- (10)  $\exists X(Xj \wedge Xp)$  (English sentence: “John has at least one thing in common with Peter.”)
- (11)  $\exists \mathcal{X}(\mathcal{X}R \wedge \mathcal{X}G)$  (English sentence: “Red has something (a property) in common with green.”)

Section 1: Recap of Lecture 5

Section 2: Historical Notes

Section 3: Beyond First-Order Logic

Section 3: The Vocabulary

Section 4: The Syntax of Second-Order Logic

Section 5: The Semantics of Second-Order Logic

Summary

References



---

## **Section 3: The Vocabulary**





## Vocabulary (shared with First-Order Logic)

The vocabulary of a second-order logic language  $L$  consists of symbols which are *shared with first-order logic languages*, and some which need to be introduced especially to fit the *second-order properties*. The once **shared with first-order logic** languages are:

- ▶ A (potentially infinite) supply of constant symbols (e.g.  $a, b, c$ , etc.),
- ▶ a (potentially infinite) supply of variable symbols representing the constants (e.g.  $x, y, z$ , etc.),
- ▶ a (potentially infinite) supply of **first-order predicate constants** (e.g.  $A, B, C$ , etc.),
- ▶ the connectives (e.g.  $\neg, \wedge, \vee, \rightarrow$ , etc.),
- ▶ the quantifiers  $\forall$  and  $\exists$ ,
- ▶ as well as the round brackets ‘(’ and ‘)’.

Section 1: Recap of Lecture 5

Section 2: Historical Notes

Section 3: Beyond First-Order Logic

Section 3: The Vocabulary

Section 4: The Syntax of Second-Order Logic

Section 5: The Semantics of Second-Order Logic

Summary

References



## Vocabulary (special to Second-Order Logic)

The vocabulary extensions to fit **second-order logic requirements** are:

- ▶ A (potentially infinite) supply of **first-order predicate variables** (e.g.  $X, Y, Z$ , etc.), which are necessary to quantify over first-order predicates,
- ▶ a (potentially infinite) supply of **second-order predicate constants** (e.g.  $\mathcal{A}, \mathcal{B}, \mathcal{C}$ , etc.).

If we wanted to take it even at a higher-order level we could also have:

- ▶ a (potentially infinite) supply of **second-order predicate variables** (e.g.  $\mathcal{X}, \mathcal{Y}, \mathcal{Z}$ , etc.) to stand in for second-order predicates.

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



## Example of a Translation Key

### Constants

j: Jumbo  
s: Simba  
b: Bambi  
m: Maya

### First-Order Pred.

$B_1x$ : x is a bee  
 $Ex$ : x is an elephant  
 $Lx$ : x is a lion  
 $Dx$ : x is a deer  
 $B_2$ : x has big ears  
 $Fx$ : x is fast  
 $Gx$ : x is gray  
 $Yx$ : x is yellow  
 $B_3$ : x is brown  
 $Cxy$ : x chases y

### Second-Order Pred.

$\mathcal{A}X$ : X is a property with  
the property of being an  
animal  
 $\mathcal{C}X$ : X is a property with  
the property of being a  
color

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



## **Section 4: The Syntax of Second-Order Logic**



# The Syntax: Recursive Definition

Given the vocabulary of  $L$  we then define the following clauses to create formulas of  $L$ :

- (i) If  $A$  is an  $n$ -ary **first-order** predicate letter/constant in  $L$ , and  $t_1, \dots, t_n$  are individual terms in  $L$ , then  $At_1, \dots, t_n$  is an (atomic) formula in  $L$ ;
- (ii) If  $X$  is a [**first-order**] predicate variable and  $t$  is an individual term in  $L$ , then  $Xt$  is an atomic formula in  $L$ ;
- (iii) If  $\mathcal{A}$  is an  $n$ -ary **second-order** predicate letter/constant in  $L$ , and  $T_1, \dots, T_n$  are **first-order unary** predicate constants, or predicate variables, in  $L$ , then  $\mathcal{A}T_1, \dots, T_n$  is an (atomic) formula in  $L$ ;
- (iv) If  $\phi$  is a formula in  $L$ , then  $\neg\phi$  is too;
- (v) If  $\phi$  and  $\psi$  are formulas in  $L$ , then  $(\phi \wedge \psi)$ ,  $(\phi \vee \psi)$ ,  $(\phi \rightarrow \psi)$ , and  $(\phi \leftrightarrow \psi)$  are too.

Section 1: Recap of Lecture 5

Section 2: Historical Notes

Section 3: Beyond First-Order Logic

Section 3: The Vocabulary

Section 4: The Syntax of Second-Order Logic

Section 5: The Semantics of Second-Order Logic

Summary

References



## The Syntax: Recursive Definition

- (vi) If  $x$  is an individual variable  $\phi$  is a formula in  $L$ , then  $\forall x\phi$  and  $\exists x\phi$  are also formulas in  $L$ ;
- (vii) If  $X$  is a [first-order] predicate variable, and  $\phi$  is a formula in  $L$ , then  $\forall X\phi$  and  $\exists X\phi$  are also formulas in  $L$ ;
- (viii) Only that which can be generated by the clauses (i)-(vii) in a finite number of steps is a formula in  $L$ .

Gamut, L.T.F (1991). Volume 1, p. 170.

**Note:** In the above clauses (i) and (ii), the word “term” is used, which has not been defined by us before. In the context here, suffices to say that it includes both constants and variables (of constants), i.e.  $a, b, c$ , etc. and  $x, y, z$ , etc.

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



## Examples of **Valid** and **Invalid** Formulas

Formula	Rule Applied
$Aa$ ✓	(i)
$Ax$ ✓	(i)
$Axy$ ✓	(i)
$Xa$ ✓	(ii)
$Xx$ ✓	(ii)
$\mathcal{A}A$ ✓	(iii)
$Xa \rightarrow \neg Xb$ ✓	(ii), (iv) and (v)
$\forall X \forall x (Xa \rightarrow Axy)$ ✓	(i), (ii), (v), (vi), and (vii)
$x$ ✗	—
$X$ ✗	—
$Xab$ ✗	—
$\forall (Xa)$ ✗	—

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



---

## **Section 5: The Semantics of Second-Order Logic**





## The Semantics of Second-Order Logic

Similar as for the syntax of second-order logic, its semantics can also be defined based on what has been defined for first-order logic before.

For instance, just as a **first-order predicate** denotes a **set of entities**, a **second-order predicate** denotes a **set of a set of entities**.

However, since the formal definitions of valuation functions get increasingly more complex, and the interpretation with regards to natural language examples more abstract, we will not further delve into the issue here.

Gamut, L.T.F (1991). Volume 1, p. 173-174.

Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

Summary

References



---

# Summary



## Summary

- ▶ **Second-order predicate logic** goes beyond first-order predicate logic by, firstly, introducing **predicate variables**, which allow to quantify over first-order predicates, and secondly, by introducing **second order predicates**, which are to be seen as properties of properties, i.e. predicates over predicates.
- ▶ These changes lead to **adjustments in the formal definitions** of the syntax and semantics of the logical language  $L$ .
- ▶ These adjustments enable the translation of a wider array of natural language sentences, although there are still natural language phenomena not captured appropriately.

Section 1: Recap of Lecture 5

Section 2: Historical Notes

Section 3: Beyond First-Order Logic

Section 3: The Vocabulary

Section 4: The Syntax of Second-Order Logic

Section 5: The Semantics of Second-Order Logic

Summary

References



# Feedback



Section 1: Recap  
of Lecture 5

Section 2:  
Historical Notes

Section 3:  
Beyond  
First-Order Logic

Section 3: The  
Vocabulary

Section 4: The  
Syntax of  
Second-Order  
Logic

Section 5: The  
Semantics of  
Second-Order  
Logic

**Summary**

References



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## References



## References

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Section 1: Recap  
of Lecture 5

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Summary

References



# Thank You.

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